Chung-Ang University

Linear Algebra Spring 2014

Problem Set #9

Assigned: May 22, 2014
Due Date: May 29, 2014

Reading: Anton, Sections 5,1 - 5,4 (pp. 295 - 327).

Problems: Given below are two sets of problems. The *Practice Problems* are for extra practice and you **do not** need to turn these in for grading. The starred problem, are the *Regular Problems* that are to be turned in for grading.

Practice Problems (Not to be turned in)

Problem 9.1

Suppose that $\mathbf{v}$ is an eigenvector of an $n \times n$ matrix $\mathbf{A}$, and let λ be the corresponding eigenvalue. Is $\mathbf{v}$ an eigenvector of the matrix

$$\mathbf{B} = \mathbf{A} - 2\mathbf{I}$$

If so, what is the eigenvalue corresponding to $\mathbf{v}$?

Problem 9.2

Without using MATLAB , find the eigenvalues and eigenvectors of the following matrix,

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

Problem 9.3

(a) Prove that $\mathbf{A}$ and $\mathbf{A}^T$ have the same eigenvalues. *Hint:* Look at the characteristic polynomials of the two matrices.

(b) Are the eigenvectors for $\mathbf{A}$ and $\mathbf{A}^T$ the same? Explain or show why or why not.

Problem 9.4

(a) **True** or **False**: If λ is an eigenvalue of $\mathbf{A}$ and if μ is an eigenvalue of $\mathbf{B}$, then $\lambda\mu$ is an eigenvalue of $\mathbf{AB}$. Justify your answer.

(b) **True** or **False**: If λ is an eigenvalue of $\mathbf{A}$ and if μ is an eigenvalue of $\mathbf{B}$, then $\lambda + \mu$ is an eigenvalue of the matrix $\mathbf{A} + \mathbf{B}$. Justify your answer.

Problem 9.5

If λ_i are the eigenvalues of $\mathbf{A}$ and μ_i are the eigenvalues of the matrix $\mathbf{B}$, then what is the product of the eigenvalues of the matrix $\mathbf{AB}$?

Problem 9.6

Find the eigenvalues and a basis for the eigenspaces of $\mathbf{A}^{25}$ where

$$\mathbf{A} = \begin{bmatrix} -1 & -2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$$

Problem 9.7

Suppose that matrices $\mathbf{A}$ and $\mathbf{B}$ are similar,

$$\mathbf{B} = \mathbf{P}^{-1}\mathbf{A}\mathbf{P}$$

Determine whether or not each of the following statements are true or false, and give a proof or justification for your answer.

(a) Similar matrices $\mathbf{A}$ and $\mathbf{B}$ have the same rank.

(b) Similar matrices $\mathbf{A}$ and $\mathbf{B}$ have the same trace.

(c) Similar matrices $\mathbf{A}$ and $\mathbf{B}$ have the same nullity.

Problem 9.8

The characteristic polynomial of a matrix $\mathbf{A}$ is

$$p(\lambda) = \lambda^4 - 3\lambda^3 + \lambda^2 + 6\lambda + 12$$

What is the determinant of $\mathbf{A}$?

Regular Problems (To be turned in for grading)

Problem 9.9★

If $\mathbf{A}$ is an invertible matrix and $\mathbf{v}$ is an eigenvector of $\mathbf{A}$, is $\mathbf{v}$ an eigenvector of $\mathbf{A}^{-1}$? If so, show why it is an eigenvector. If not, explain why it is not an eigenvector.

Problem 9.10★

Let $\mathbf{A}$ be a matrix of ones and $\mathbf{B}$ a checkerboard matrix,

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

Without using MATLAB

(a) Find the rank of each matrix.

(b) Find the eigenvalues and eigenvectors of $\mathbf{A}$ and $\mathbf{B}$.

Problem 9.11★

Find a matrix $\mathbf{A}$ whose eigenvalues are $\lambda_1 = 1$ and $\lambda_2 = 4$, and eigenvectors

$$\mathbf{v}_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \quad ; \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Problem 9.12★

Find a basis for the eigenspaces of the following matrices

(a) $\mathbf{A}_1 = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\mathbf{A}_2 = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$ (b) $\mathbf{A}_3 = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$

Problem 9.13★

Find the matrix $\mathbf{P}$ that diagonalizes each of the following matrices,

$$\text{(a)}\ \mathbf{A}_1 = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \qquad \text{(b)}\ \mathbf{A}_2 = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$