Chung-Ang University

Linear Algebra Spring 2014

Problem Set #7

Assigned: May 01, 2014
Due Date: May 13, 2014

Reading: Anton, Sections 4.7 (pp. 225-247).

Problems: Given below are two sets of problems. The *Practice Problems* are for extra practice and you **do not** need to turn these in for grading. The starred problem, are the *Regular Problems* that are to be turned in for grading.

Practice Problems (Not to be turned in)

Problem 7.1

Let $\mathbf{A}$ be an $m \times n$ matrix. What is the smallest possible value for its nullity?

Problem 7.2

(a) Give an example of a 3×3 matrix whose column space is a two-dimensional plane through the origin in R^3.

(b) What type of geometric object is the null space of your matrix in part (a)?

(c) What type of geometric object is the row space of your matrix in part (a)?

Problem 7.3

True or **False**: If $\mathbf{A}$ is an $n \times n$ matrix, then the nullspace is equal to the left nullspace.

Problem 7.4

Find the dimension and construct a basis for the four subspaces associated with the following matrix,

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Problem 7.5

What is the dimension of the nullspace of the 4×4 identity matrix?

Regular Problems (To be turned in for grading)

Problem 7.1⋆

Show that if the product of two matrices is the zero matrix, $\mathbf{AB} = \mathbf{0}$, then the column space of $\mathbf{B}$ is *contained* in the nullspace of $\mathbf{A}$.

Problem 7.2⋆

Construct a matrix whose nullspace is spanned by $[1,\ 0,\ 1]^T$.

Problem 7.3⋆

Listed below is the output of MATLAB showing the row echelon form $\mathbf{R}$ of a matrix $\mathbf{A}$:

```
A =
     2     1     5     8     4     2     2
     1     0     2     4     1     1     1
    -1    -3    -5    -4    -7     1     1
     1     0     2     5     3     1     0
    -1    -2    -4    -1     1     3     0

R =
     1     0     2     0    -7     0     4
     0     1     1     0     2     0     0
     0     0     0     1     2     0    -1
     0     0     0     0     0     1     1
     0     0     0     0     0     0     0
```

Find the dimension and construct a basis for each of the four subspaces that are associated with the matrix $\mathbf{A}$.

Problem 7.4⋆

For each of the following problems, determine whether or not the linear system

$$\mathbf{Ax} = \mathbf{b}$$

is consistent and, if it is, find the dimension of the solution set, which is equivalent to the number of *free parameters*.

(a) $\mathbf{A}$ is 4×4 and $\text{Rank}(\mathbf{A}) = 4$, $\text{Rank}([\mathbf{A}; \mathbf{b}]) = 4$.

(b) $\mathbf{A}$ is 5×5 and $\text{Rank}(\mathbf{A}) = 3$, $\text{Rank}([\mathbf{A}; \mathbf{b}]) = 4$.

(c) $\mathbf{A}$ is 6×6 and $\text{Rank}(\mathbf{A}) = 3$, $\text{Rank}([\mathbf{A}; \mathbf{b}]) = 3$.

(d) $\mathbf{A}$ is 5×7 and $\text{Rank}(\mathbf{A}) = 3$, $\text{Rank}([\mathbf{A}; \mathbf{b}]) = 4$.

(e) $\mathbf{A}$ is 7×4 and $\text{Rank}(\mathbf{A}) = 3$, $\text{Rank}([\mathbf{A}; \mathbf{b}]) = 3$.

Problem 7.5⋆

Given two matrices $\mathbf{A}$ and $\mathbf{B}$, what is the relationship between the subspaces given below? In other words, are they equal, is one subspace contained within the other, or is there no relationship at all?

(a) $\mathcal{N}(\mathbf{AB})$ and $\mathcal{N}(\mathbf{B})$.

(b) $\mathcal{R}(\mathbf{AB})$ and $\mathcal{R}(\mathbf{A})$.

(c) $\mathcal{N}((\mathbf{AB})^T)$ and $\mathcal{N}(\mathbf{A}^T)$.

(d) $\mathcal{R}((\mathbf{AB})^T)$ and $\mathcal{R}(\mathbf{B}^T)$.

Problem 7.6⋆

Given two matrices $\mathbf{A}$ and $\mathbf{B}$, what is the relationship between

(a) The rank of $\mathbf{AB}$ and the rank of $\mathbf{A}$.

(b) The rank of $\mathbf{AB}$ and the rank of $\mathbf{B}$.

(c) The dimension of the nullspace of $\mathbf{AB}$ and the dimension of the nullspace of $\mathbf{B}$