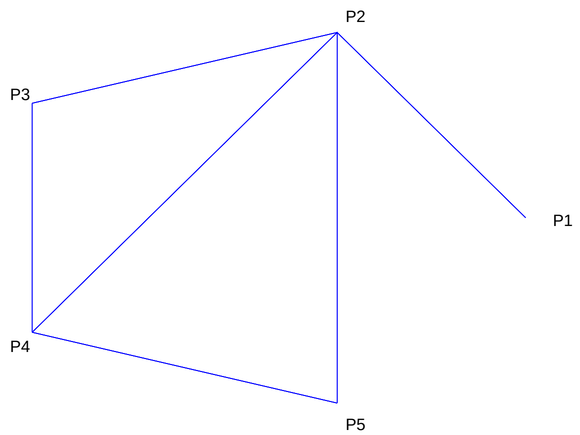
CHUNG-ANG UNIVERSITY

Linear Algebra   Spring 2014

Solutions to Computer Project #2

Problem 2.1

Find the adjacency matrix for the following graph (all edges are bidirectional).



Solution

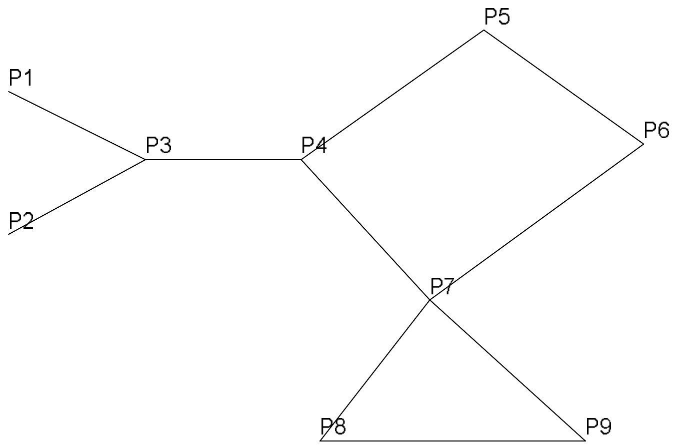
From the given bidirectional graph we see that the adjacency matrix is

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

As a check, make sure the matrix is *symmetric* and make sure the number of entries that are equal to one is equal to two times the number of edges in the graph (there are six edges). You may also run the m-file `graph.m` in MATLAB or Octave to verify that you entered the adjacency matrix correctly.

Problem 2.2

Find the number of paths of length nine from P_1 to P_9 in the following bidirectional graph.



Solution

From the the following bidirectional graph we want to find the number of paths of length nine from P_1 to P_9. The first thing we do is find the adjacency matrix, which is

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

To find the number of length nine paths from one vertex to another, we form the matrix $\mathbf{A}^9$, which in this case we find using Octave is equal to

```
octave:22> B^9
ans =

     2     2   253    34   261    32   411   103   103
     2     2   253    34   261    32   411   103   103
   253   253    38   925    66   672   272   514   514
    34    34   925   308  1075   274  1803   580   580
   261   261    66  1075   112   814   404   653   653
    32    32   672   274   814   242  1392   477   477
   411   411   272  1803   404  1392  1066  1216  1216
   103   103   514   580   653   477  1216   612   613
   103   103   514   580   653   477  1216   613   612
```

Therefore, if we look at the last entry in the first row we will see the number of paths of length nine from P_1 to P_9.

Problem 2.3

Let $\mathbf{A}$ be an 8×8 adjacency matrix for a graph with $a_{ij} = 1$ if $i \neq j$ and $a_{jj} = 0$ for all j. In other words, all the elements of $\mathbf{A}$ are equal to one except along the main diagonal. In MATLAB , this matrix would be given by

```
>> A = ones(8,8) - eye(8)
```

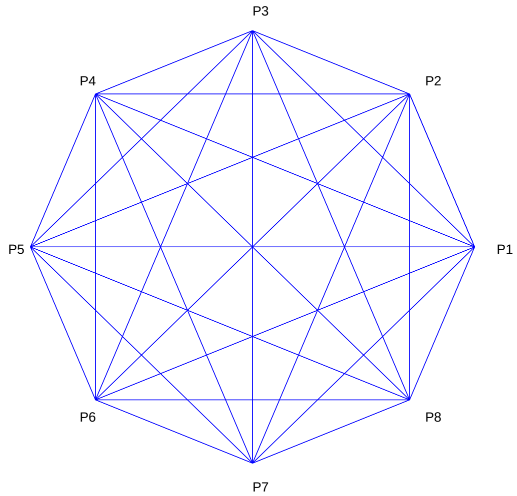
(a) Draw the graph associated with this adjacency matrix.

(b) How may paths of length four are there from node 1 to node 4?

Solution

We now have an adjacency matrix for a graph with eight nodes with the property that $a_{ij} = 1$ if $i \neq j$ and $a_{jj} = 0$ for all j. What this means is that every node is connected to every other node.

(a) To draw the graph, we draw eight nodes on a piece of paper (the location or the pattern does not matter), and we connect every point to the other with an edge. If we arrange the eight nodes in a circle, we would have a graph as shown in the figure below, which was drawn using the `graph.m` m-file.



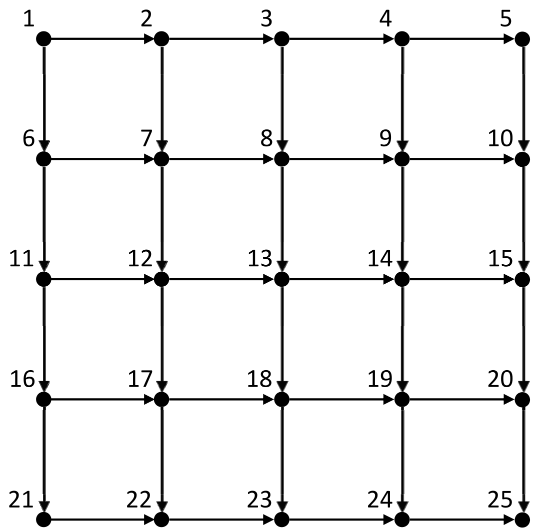
(b) We may find the number of paths of length four node 1 to node 4 in Octave as follows

```
>> A = ones(8,8) - eye(8)
>> A^4
ans =
     301    300    300    300    300    300    300    300
     300    301    300    300    300    300    300    300
     300    300    301    300    300    300    300    300
     300    300    300    301    300    300    300    300
     300    300    300    300    301    300    300    300
     300    300    300    300    300    301    300    300
     300    300    300    300    300    300    301    300
     300    300    300    300    300    300    300    301
```

Therefore, the number of paths of length four from any node to any *other* node is equal to 300.

Problem 2.4

All of the streets in downtown Seoulville run one-way, either west to east or north to south as indicated in the graph below. Therefore, traffic can only move to the right and down the grid of streets. The nodes of the graphs are intersections.



(a) Create the adjacency matrix **A** for this graph (digraph) and enter it into MATLAB . *Hint:* The adjacency matrix is *sparse* in this case. In other words, most of the elements of **A** are equal to zero. Therefore, it is easier to create the adjacency matrix as follows (only the first few entries are shown)

```
>> A = zeros(25);
>> A([2 6],1)=[1 ; 1];
>> A([3 7],2)=[1 ; 1];
```

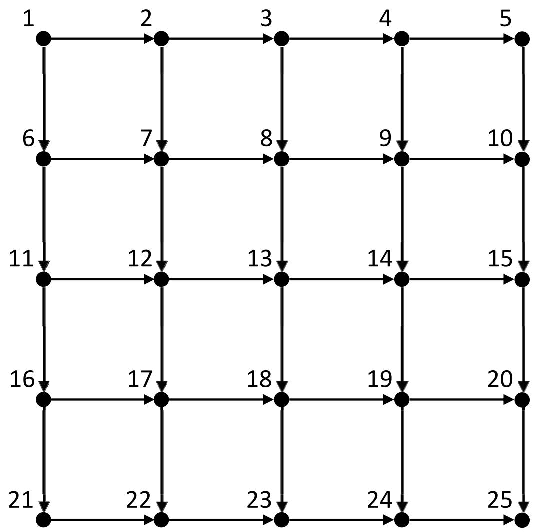
You may also want to use your MATLAB text editor and type in the commands to generate **A** and then save it as an m-file that you can run (this allows for simple correction of errors, if you make any).

(b) How many paths of length eight are there from intersection 1 to intersection 25, which is the shortest distance between these intersections?

(c) Suppose that there is a water main break on the streets connecting intersection 8 to intersection 18 so that there is no access to these street. Also, there in construction on the road connection intersections 4 and 5, so this road is inaccessible. How many paths of length 8 are there now from intersection 1 to intersection 25?

Solution

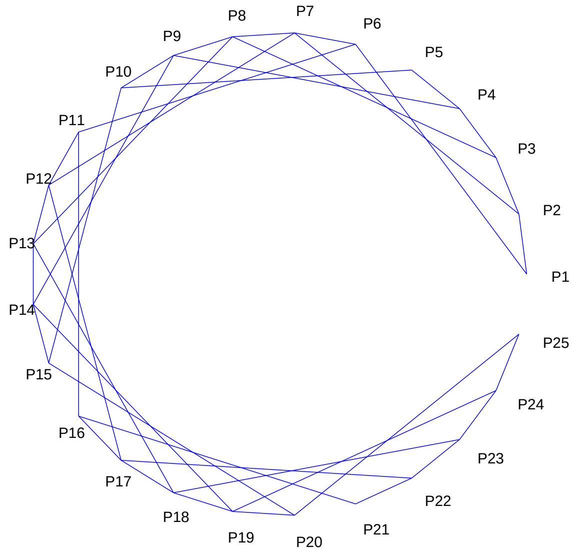
We are given the following directed graph (di-graph) consisting of twenty-five nodes.



(a) The adjacency matrix for this graph may be created as following using MATLAB or Octave by writing a program similar to the following (the syntax will be slightly different for MATLAB),

```
function [A] = seoulville ()
A = zeros(25);
for j=1:5
for i=1:4
     A(i+5*(j-1),i+5*(j-1)+1)=1;
     A(j+5*(i-1),j+5*(i-1)+5)=1;
     for;
for;
```

We can verify that the adjacency matrix was generated correctly by making a graph from **A**. Using the m-file `graph.m` we obtain the graph shown below,



Although the nodes of the graph are laid out differently, we can see that all of the edges are generated correctly and, by looking at the matrix, we can verify that the directed edges are pointing in the right direction (all entries that are non-zero are above the main diagonal).

(b) To find the number of paths of length eight from intersection 1 (node 1) to intersection 25 (node 25) we form the matrix $\mathbf{B} = \mathbf{A}^8$. From the entry $b_{1,25}$ we find that there are 70 paths.

(c) If the path connecting intersection 8 to intersection 18 is closed, we set $A(8, 13) = A(13, 18 + 0$, and if there is construction on the road connection intersections 4 and 5, then we set $A(4, 5) = 0$. To find out how many paths of length eight remain, we repeat what we did in part (b) and we find that there are now only 42.

Problem 2.5

The dominance matrix for the 2001 Southeastern Conference (American) Football season is given below,

$$\mathbf{S} = \begin{array}{cl} \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} & \begin{array}{l} \text{Alabama} \\ \text{Arkansas} \\ \text{Auburn} \\ \text{Florida} \\ \text{Georgia} \\ \text{Kentucky} \\ \text{Louisiana State} \\ \text{Missippi} \\ \text{Missippi State} \\ \text{South Carolina} \\ \text{Tennesee} \\ \text{Vanderbilt} \end{array} \end{array}$$

The names of the teams for each of the rows are indicated next to the dominance matrix.

(a) Find the number of victories for each team and the power ranking of each team.

(b) Rank order the teams in terms of the number of victories and power.

(c) Auburn, Georgia, Louisiana State, and South Carolina all have the same number of victories. Which team has the highest power rating? Which one has the lowest?

Solution

The dominance matrix for the 2001 Southeastern Conference (American) Football season is given below,

$$\mathbf{S} = \begin{array}{cl} \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} & \begin{array}{l} \text{Alabama} \\ \text{Arkansas} \\ \text{Auburn} \\ \text{Florida} \\ \text{Georgia} \\ \text{Kentucky} \\ \text{Louisiana State} \\ \text{Missippi} \\ \text{Missippi State} \\ \text{South Carolina} \\ \text{Tennesee} \\ \text{Vanderbilt} \end{array} \end{array}$$

(a) The dominance matrix for this team, S, is given in the data set for this problem set. Loading the appropriate on into MATLAB or Octave, we can generate the number of victories either by counting the number of ones in each row or with the following command

```
>> v = S*ones(12,1);        %Number of victories for each team in v
```

and we may find the power ranking with the following command

```
>> p = (S+S^2)*ones(12,1); %Power ranking for each team in p
```

What we find for both is given in the table below

Team	Victories	Power Ranking
Alabama	4	15
Arkansas	4	20
Auburn	5	22
Florida	6	24
Georgia	5	21
Kentucky	1	1
Louisiana State	5	21
Mississippi	4	14
Mississippi State	2	7
South Carolina	5	17
Tennessee	7	32
Vanderbilt	0	0

We see that the top three teams in terms of their power ranking are Tennessee (32), Florida (24), and Auburn (22).

(b) Auburn, Georgia, Louisiana State, and South Carolina all have the same number of victories, five. Of these teams, Auburn has the highest power ranking and South Carolina has the smallest.