Chung-Ang University

Linear Algebra Spring 2014

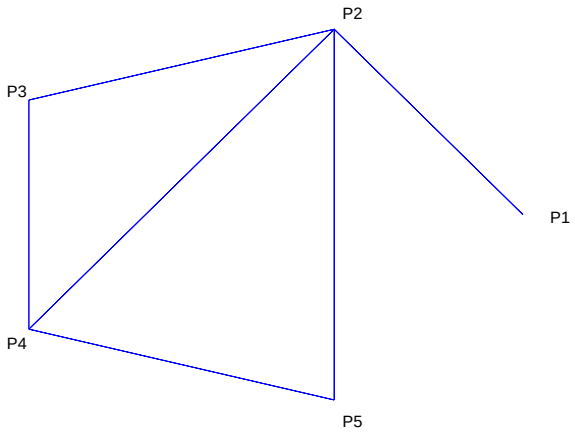
Computer Project #2

Assigned: April 29, 2014
Due Date: May 13, 2014

Data File: Included with this problem set is a MATLAB and Octave data file that contains the adjacency matrix for Problem 2.5. Also included is a nice MATLAB GUI named `graph_gui.m` for drawing graphs. Read the documentation for how to use it. Unfortunately, this GUI will not work in Octave. Also included is an m-file that will work both in MATLAB and Octave that will take an adjacency matrix and draw the graph. Note, however, that this routine assumes a unidirectional graph, and will not draw directional paths.

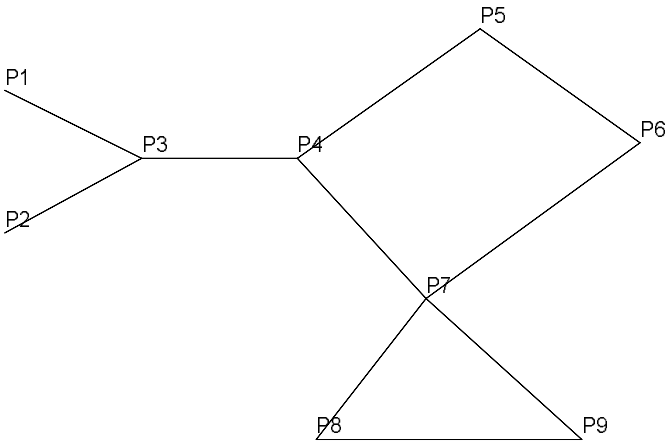
Problem 2.1

Find the adjacency matrix for the following graph (all edges are bidirectional).



Problem 2.2

Find the number of paths of length nine from P_1 to P_9 in the following bidirectional graph.



Problem 2.3

Let $\mathbf{A}$ be an 8×8 adjacency matrix for a graph with $a_{ij} = 1$ if $i \neq j$ and $a_{jj} = 0$ for all j. In other words, all the elements of $\mathbf{A}$ are equal to one except along the main diagonal. In MATLAB , this matrix would be given by

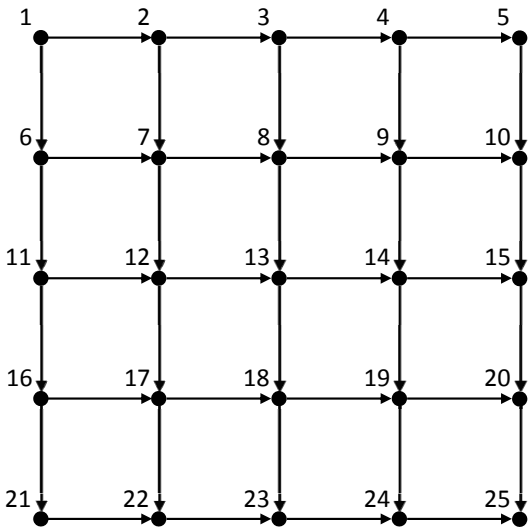
```
>> A = ones(8,8) - eye(8)
```

(a) Draw the graph associated with this adjacency matrix.

(b) How may paths of length four are there from node 1 to node 4?

Problem 2.4

All of the streets in downtown Seoulville run one-way, either west to east or north to south as indicated in the graph below. Therefore, traffic can only move to the right and down the grid of streets. The nodes of the graphs are intersections.



(a) Create the adjacency matrix $\mathbf{A}$ for this graph (digraph) and enter it into MATLAB . *Hint:* The adjacency matrix is *sparse* in this case. In other words, most of the elements of $\mathbf{A}$ are equal to zero. Therefore, it is easier to create the adjacency matrix as follows (only the first few entries are shown)

```
>> A = zeros(25);
>> A([2 6],1)=[1 ; 1];
>> A([3 7],2)=[1 ; 1];
```

You may also want to use your MATLAB text editor and type in the commands to generate $\mathbf{A}$ and then save it as an m-file that you can run (this allows for simple correction of errors, if you make any).

(b) How many paths of length eight are there from intersection 1 to intersection 25, which is the shortest distance between these intersections?

(c) Suppose that there is a water main break on the streets connecting intersection 8 to intersection 18 so that there is no access to these street. Also, there in construction on the road connection intersections 4 and 5, so this road is inaccessible. How many paths of length 8 are there now from intersection 1 to intersection 25?

Problem 2.5

The dominance matrix for the 2001 Southeastern Conference (American) Football season is given below,

$$\mathbf{S} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \text{Alabama} \\ \text{Arkansas} \\ \text{Auburn} \\ \text{Florida} \\ \text{Georgia} \\ \text{Kentucky} \\ \text{Louisiana State} \\ \text{Missippi} \\ \text{Missippi State} \\ \text{South Carolina} \\ \text{Tennesee} \\ \text{Vanderbilt} \end{matrix}$$

The names of the teams for each of the rows are indicated next to the dominance matrix.

(a) Find the number of victories for each team and the power ranking of each team.

(b) Rank order the teams in terms of the number of victories and power.

(c) Auburn, Georgia, Louisiana State, and South Carolina all have the same number of victories. Which team has the highest power rating? Which one has the lowest?