Chung-Ang University

Linear Algebra Spring 2014

Problem Set #5

Assigned: April 08, 2014
Due Date: April 15, 2014

Reading: Chapter 4 Sections 4.1 - 4.5 in Anton, pp. 171-215.

This is a **very important** set of problems because they will help you understand the important concepts of linear independence, basis, and dimension. In these problems, you are asked to think about vector spaces that are not the normal Euclidean spaces, R^n, that we have been working with, but rather vector spaces of polynomial, matrices, functions and others.

The following problems are short answer and do not require much time or effort. You must, however, give a reason for each of your answers.

Problem 5.1

Which of the following are *subspaces* of R^4 (give a reason for your answer).

(a) The set of all vectors of the form $[a,\ 0,\ a,\ 0]^T$.

(b) The set of all vectors of the form $[a,\ b,\ a+b,\ a-b]^T$.

(c) The set of all vectors of the form $[a,\ 0,\ b,\ 0]^T$.

(d) The set of all vectors of the form $[1,\ 0,\ a,\ 0]^T$.

Problem 5.2

Which of the following are *subspaces* of R^∞ (give a reason for your answer).

(a) The set of all sequences $\mathbf{v}$ in R^∞ that are of the form

$$\mathbf{v} = [a,\ 0,\ a,\ 0,\ a,\ 0\cdots]^T$$

(b) The set of all sequences $\mathbf{v}$ in R^∞ that are of the form

$$\mathbf{v} = [a,\ 1,\ a,\ 1,\ a,\ 1\cdots]^T$$

(c) The set of all sequences $\mathbf{v}$ in R^∞ that are of the form

$$\mathbf{v} = [a,\ 2a,\ 4a,\ 8a,\ 16a,\cdots,\ 2^n a,\cdots]^T$$

(d) The set of all sequences in R^∞ whose components are equal to zero from some point on.

Problem 5.3

Which of the following are *subspaces* of $M_{n\times n}$ (give a reason for your answer).

(a) The set of all invertible matrices.

(b) The set of all diagonal matrices.

Problem 5.4

Which of the following are *subspaces* of P_4, the vector space of all polynomials of degree 4 or less.

(a) All polynomials of the form $p(x) = ax$.

(b) All polynomials that have real roots.

Problem 5.5

What is the dimension of the following vector spaces?

(a) The vector space of all $n \times n$ upper triangular matrices

(b) All vectors of the form $[a, b, 0, 0]^T$

(c) All polynomials of the form $p(x) = ax^2 + bx^4$.

(d) The vector space of all 2×2 matrices

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

where $a + b + c = 0$.

Problem 5.6

Which of the following sets of vectors form a basis for R^3?

(a) $\mathbf{v}_1 = [1,\ 0,\ 0]^T$, $\mathbf{v}_2 = [1,\ 1,\ 0]^T$, $\mathbf{v}_3 = [1,\ 1,\ 1]^T$.

(b) $\mathbf{v}_1 = [1,\ 0,\ 0]^T$, $\mathbf{v}_2 = [0,\ 1,\ 0]^T$, $\mathbf{v}_3 = [0,\ 0,\ 1]^T$, $\mathbf{v}_4 = [1,\ 0,\ 1]^T$.

(c) $\mathbf{v}_1 = [1,\ 2,\ 3]^T$, $\mathbf{v}_2 = [3,\ 2,\ 1]^T$, $\mathbf{v}_3 = [1,\ 2,\ 4]^T$

Problem 5.7

Which of the following vectors in $\mathbf{P}_3$ are linearly independent? For those sets that are linearly dependent, which vectors in the set are dependent upon the others?

(a) $\mathbf{p}_1(x) = 1,\ \ \mathbf{p}_2(x) = 1 + x,\ \ \mathbf{p}_3(x) = 1 + x + x^2,\ \ \mathbf{p}_4(x) = 1 + x + x^2 + x^3.$

(b) $\mathbf{p}_1(x) = 1 + x,\ \ \mathbf{p}_2(x) = 1 - x,\ \ \mathbf{p}_3(x) = 1 + x^2 + x^3.$

(c) $\mathbf{p}_1(x) = 1,\ \ \mathbf{p}_2(x) = 1 + x,\ \ \mathbf{p}_3(x) = 1 + x^2,\ \ \mathbf{p}_4(x) = x^3.$

Problem 5.8

You are given a set of ten vectors in R^{20}. How would you use MATLAB to determine whether or not these vectors are linearly independent?