Chung-Ang University

Linear Algebra Spring 2014

Problem Set #4

Assigned: March 27, 2014
Due Date: April 03, 2014

Reading: Anton, Sections 3.1 - 3.4 (pp. 119-160).

Problems: Given below are two sets of problems. The *Practice Problems* are for extra practice and you **do not** need to turn these in for grading. The starred problem, are the *Regular Problems* that are to be turned in for grading.

Practice Problems (Not to be turned in)

Problem 4.1

For what values of α, if any, is the vector $\mathbf{v}^T = [8\alpha,\ -2]$ parallel to the vector $\mathbf{u}^T = [4,\ -1]$?

Problem 4.2

Let $\mathbf{u}$ and $\mathbf{v}$ be the following vectors:

$$\mathbf{u} = [\ 2,\ 1,\ 0,\ 1,\ -1]^T; \quad \mathbf{v} = [-2,\ 3,\ 1,\ 0,\ \ 2]^T$$

and let

$$\mathbf{c} = [-8,\ 8,\ 3,\ -1,\ 7]^T$$

Find scalars a and b such that

$$a\mathbf{u} + b\mathbf{v} = \mathbf{c}$$

Problem 4.3

Let P be the point $(2,\ 3,\ -2)$ and Q the point $(7,\ -4,\ 1)$.

(a) Find the midpoint of the line segment connecting P and Q.

(b) Find the point on the line segment connecting P and Q that is 3/4 of the way from P to Q.

Problem 4.4

Let $\mathbf{u}$ and $\mathbf{v}$ be the following vectors,

$$\mathbf{u} = [\ 2,\ -1,\ -4,\ 1,\ 0,\ 6,\ -3,\ 1]^T$$
$$\mathbf{v} = [-2,\ -1,\ 2,\ 3,\ 7,\ 2,\ -5,\ 1]^T$$

(a) Find the (Euclidean) distance between $\mathbf{u}$ and $\mathbf{v}$.

(b) Find the cosine of the angle between $\mathbf{u}$ and $\mathbf{v}$.

(c) Find a vector that is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$.

Regular Problems (To be turned in for grading)

Problem 4.1★

Find a unit vector that is orthogonal to the vectors

$$\mathbf{u} = [\ 1,\ 2,\ -1,\ 3]^T; \quad \mathbf{v} = [-1,\ 0,\ 4,\ -1]^T$$

Problem 4.2★

Find a point-normal form equation for a plane that passes through the point

$$P = (1,\ 1,\ 4)$$

and is normal to the vector

$$\mathbf{n} = [\ 1,\ 9,\ 8]^T$$

Problem 4.3

In each part below you are given a pair of linear equations that represent planes in R^3. For each, determine whether or not the planes are parallel, orthogonal, or neither.

(a)

$$\begin{aligned} x - \ 4y - 3z &= 2 \\ 3x - 12y - 9z &= 7 \end{aligned}$$

(b)

$$\begin{aligned} 2x + 3y - \ z &= 6 \\ x - 4y + 4z &= 1 \end{aligned}$$

(c)

$$\begin{aligned} 3x + \ y - 2z &= 6 \\ 2x - 4y + \ z &= 0 \end{aligned}$$

Problem 4.4★

Find $\|\text{proj}_{\mathbf{a}}\mathbf{u}\|$, the projection of $\mathbf{u}$ onto the vector $\mathbf{a}$, when

$$\mathbf{u} = [1,\ -1,\ 0]^T; \quad \mathbf{a} = [2,\ 0,\ 1]^T$$

Problem 4.5★

Find the component of the vector $\mathbf{u}$ along the vector $\mathbf{a}$ and the vector component of $\mathbf{u}$ that is orthogonal to $\mathbf{a}$ when

(a) $\mathbf{u} = [3,\ -1,\ 2]^T$ and $\mathbf{a} = [1,\ 3,\ 0]^T$.

(b) $\mathbf{u} = [5,\ 0,\ -3,\ 7]^T$ and $\mathbf{a} = [2,\ 1,\ -1,\ -1]^T$.

Problem 4.6★

Find the distance between the point $P = (2,\ 1,\ 3)$ and the plane

$$3x - y - 2z = 6$$

Problem 4.7★

Find the distance between the two parallel planes

$$\begin{aligned} 2x - \ y - \ z &= 5 \\ -4x + 2y + 2z &= 12 \end{aligned}$$