# Chung-Ang University

Linear Algebra Spring 2014

## Problem Set #1

Assigned: March 04, 2014
Due Date: March 13, 2014

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**Reading:** Anton, Sections 1.1 and 1.2 (pp. 1-24).

**Problems**: Given below are two sets of problems. The *Practice Problems* are for extra practice and you **do not** need to turn these in for grading. The starred problem, are the *Regular Problems* that are to be turned in for grading. Generally, only answers will be given for the optional problems, whereas detailed solutions will be provided for the required problems.

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## Practice Problems (Not to be turned in)

### Problem 1.1

Write a system of three linear equations in three unknowns that have:

(a) A unique solution,

(b) No solution,

(c) An infinite number of solutions.

### Problem 1.2

Find the augmented matrix for each of the following linear systems,

(a)
$$\begin{aligned} -2x_1 &= 6 \\ 3x_1 &= 8 \\ 9x_1 &= -3 \end{aligned}$$

(b)
$$\begin{aligned} 2x_2 - 3x_4 + x_5 &= 0 \\ -3x_1 - x_2 + x_3 &= -1 \\ 6x_1 + 2x_2 - x_3 + 2x_4 - 3x_5 &= 6 \end{aligned}$$

(c)
$$\begin{aligned} x_1 - x_3 &= 4 \\ x_2 + x_4 &= 2 \end{aligned}$$

(d)
$$\begin{aligned} x_1 - x_3 &= 4 \\ x_2 + x_4 &= 2 \end{aligned}$$

### Problem 1.3

A pharmacist needs eight ounces of a solution that is 50% saline. What he has is a bottle of 60% saline solution and a bottle of 20% saline solution. How many ounces of 60% saline solution and 20% saline solution must be mixed to obtain the mixture needed?

## Problem 1.4

1. (True or False): If the reduced echelon form of the augmented matrix for a linear system has a row of zeros, then the system must have an infinite number of solutions.
2. (True or False): If a linear system has more unknowns than equations, then it must have infinitely many solutions.
3. (True or False): A homogeneous linear system will have a unique solution.

## Problem 1.5

If $\mathbf{A}$ is an $n \times n$ matrix, how many different forms can the reduced row echelon for of the matrix have? Give an example of each.

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## Regular Problems (To be turned in for grading)

## Problem 1.1⋆

For each of the following row echelon forms of an augmented matrix for a linear system, find the solution to the linear equations.

(a) $\begin{bmatrix} 1 & -3 & 4 & 7 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 5 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & 8 & -5 & 6 \\ 0 & 1 & 4 & -9 & 3 \\ 0 & 0 & 1 & 1 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 7 & -2 & 0 & -8 & -3 \\ 0 & 0 & 1 & 1 & 6 & 5 \\ 0 & 0 & 0 & 1 & 3 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & -3 & 7 & 1 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

## Problem 1.2⋆

Solve the following linear system

$$\begin{aligned} 2x + 2y + 4z &= 2 \\ w - y - 3z &= 0 \\ 2w + 3x + y + z &= 3 \\ -2w + x + 3y - 2z &= 1 \end{aligned}$$

## Problem 1.3⋆

Solve the following linear system

$$\begin{aligned} x_1 - x_2 + 7x_3 + x_4 &= 0 \\ x_1 + 2x_2 - 6x_3 - x_4 &= 0 \end{aligned}$$

## Problem 1.4⋆

Find the coefficients $a$, $b$, $c$, and $d$ of a cubic equation

$$y = ax^3 + bx^2 + cx + d$$

that passes through the following four points, $(x, y)$:

$$(0, 10), \quad (1, 8), \quad (3, -10), \quad (4, -14)$$

This is an example of fitting a polynomial to some data (set of points) that we will be studying later. A more interesting example is when we have a polynomial, such as the one above, and we want to find the *best fitting* polynomial through a large number of points (much larger than four). This falls into the category of the very important problem of signal or data modeling.