Chapter 4
AC Network Analysis

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Capacitance
Inductance and Induction
Time-Varying Signals
Sinusoidal Signals

Reference: David K. Cheng, Field and Wave Electromagnetics.
Energy Storage Circuit Elements

- Energy loss element: resistors
- Energy storage element: capacitors and inductors (in the form of electromagnetic field)

- Ideal capacitor
- Ideal inductor

- In practice, any component of an electric circuit will exhibit phenomena of some resistance, some inductance, and some capacitance.
The Ideal Capacitor

- A physical capacitor is a device that can store energy in the form of a charge separation when appropriately polarized by an electric field, or voltage. That is, the ideal capacitors store energy (electric charges on the conducting plates) in the form of electric field.

- **Capacitance** $C$ is the measure of how much electric charge can be stored in a capacitor. It depends on material properties only.

- The simplest capacitor consists of two parallel conductors separated by a dielectric (insulator), which has very large resistances.

- The insulating material does not allow for the flow of DC current: thus, a capacitor acts as an open circuit for DC current.

- **Charging**: Applying a voltage to a (discharged) capacitor causes a current to charge the capacitor. That is, electric charges move to the capacitor, but they can’t go through the capacitor.

- **Discharging**: Connecting a path across the terminals of a charged capacitor causes current to flow (because it has energy).
Charging & Discharging

- **Charging** (left switch closed, right switch open)
  - The electric charges from the voltage source move to the capacitor, so capacitor voltage and energy increases up to $V_B$.

- **Discharging** (left switch open, right switch closed)
  - The electric charges from the capacitor move to the resistor, so the energy accumulated on the capacitor dissipates in the resistor.
The Unit of Capacitance and Energy

- The farad (F) is the unit of capacitance.
  - One farad of capacitance equals one coulomb of charge stored in the dielectric with one volt applied.
  - Most capacitors have values less than 1 F: 1 µF (microfarad) = $1 \times 10^{-6}$ F, 1 nF (nanofarad) = $1 \times 10^{-9}$ F, 1 pF (picofarad) = $1 \times 10^{-12}$ F

- Charge on a capacitor is generated due to voltage applied across the capacitor: $q = CV$

$$i_C(t) = \frac{dq_C(t)}{dt} = \frac{d[Cv_C(t)]}{dt} = C \frac{dv_C(t)}{dt} \quad \frac{dv_C(t)}{dt} = \frac{i_C(t)}{C} \rightarrow [v_C(t')]_{t = \infty} = \int_{-\infty}^{t} \frac{i_C(t')}{C} dt'$$

$$v_C(t) = v_C(t = 0) + \int_{0}^{t} \frac{i_C(t')}{C} dt'$$

$$W_C(t) = \int_{0}^{t} p(t')dt' = \int_{0}^{t} v_C(t')i_C(t')dt' = \int_{0}^{t} v_C(t')C \frac{dv_C(t')}{dt} dt'$$

$$W_C(t) = \frac{1}{2} C v_C^2(t) \text{ Energy stored in a capacitor (J)}$$

$C$ : capacitance in farads $\quad v_C$ : voltage in volts
Series and Parallel Capacitances

- Connecting capacitances in series is equivalent to increasing the distance between the conducting plates.

  \[ \frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots \text{ etc.} \]

- Total \( C \) is less than the smallest individual value.

- Connecting capacitances in parallel is equivalent to increasing plate area where can store charge.

- Total \( C \) is the sum of individual \( C \)s:

\[ C_T = C_1 + C_2 + \ldots \text{ etc.} \]

- Voltage is the same across parallel capacitors.

\[ i = i_1 + i_2 + i_3 = C_1 \frac{dv_1(t)}{dt} + C_2 \frac{dv_2(t)}{dt} + C_3 \frac{dv_3(t)}{dt} = (C_1 + C_2 + C_3) \frac{dv(t)}{dt} = C_{EQ} \frac{dv(t)}{dt} \]
Magnetic Field around an Electric Current

• A circular magnetic field is produced by the flow of current through a straight conductor in the center.

• The direction of the magnetic field inside a coil is perpendicular to the current flowing through the coil.

• The polarity of the magnetic field is based on the right-hand rule.

**Right-hand rule:**
The thumb: $B \rightarrow$ the other fingers: $i$
The thumb: $i \rightarrow$ the other fingers: $B$
Induced Current

• When a moving conductor cuts across magnetic flux lines, current is induced.
  – The polarity of induced voltage is determined by Lenz’s law.

• Lenz’s law states that the direction of an induced current must be such that its own magnetic field will oppose the change that produced the induced current. -> What if the permanent magnet does not move?

• The direction of the induced current is determined by right-hand rule for current flow. If the fingers coil around the direction of current shown, the thumb will point to the left for the north pole.

Induced current produced by magnetic flux cutting across turns of wire in a coil.
Induced Voltage

- **Faraday’s Law of Induced Voltage**
  
  The amount of voltage induced is determined by the following formula.

  \[
  v_{\text{ind}} = N \frac{d\Phi}{dt} (\text{webers} / \text{seconds})
  \]

  - \(N\) = number of turns
  - \(d\Phi/dt\) = how fast the magnetic flux cuts across the conductor

- Either the **flux** or the **conductor** should move to induce voltages.

  Voltage induced across coil cut by magnetic flux. (a) Motion of flux generating voltage across coil. (b) Induced voltage acts in series with coil. (c) *Induced voltage is a kind of voltage source that can produce current* in an external load resistor \(R_L\) connected across the coil.
Self-Induced Voltage

• Lenz’s law states that the direction of an induced current must be such that its own magnetic field will **oppose** the change that produced the induced current.

• When $i_L$ increases, $v_L$ has polarity that opposes the increase in current.
• When $i_L$ decreases, $v_L$ has polarity to oppose the decrease in current.

• In both cases, the change in current is opposed by the induced voltage.
• What if the magnitude of current is constant? (DC case)
The Ideal Inductor

- The ideal inductors store energy (electric charges on the conducting plates) in the form of magnetic field.
- A inductor is typically made by winding a coil of wire around a core (an insulator or a ferromagnetic material).

Ferromagnetic materials include iron, steel, nickel, cobalt, and certain alloys (usually conductors). They can become strongly magnetized in the same direction as the external magnetizing field.

**Inductance** $L$ is the measure of the ability of a conductor to induce voltage when the current changes or ability to store energy in a magnetic field. *It depends on material properties only.*
Example of Inductance $L$

- Inductance is a function of the number of turns ($N$), a cross sectional area ($A$), permeability of core ($\mu_r$), and the length of a core ($l$).

Calculating the Inductance of a Long Coil

\[
L = \mu_r \frac{N^2A}{l} \times 4 \pi \times 10^{-7} \text{ H}
\]

Where:
- $L$ is the inductance in henrys.
- $\mu_r$ is the relative permeability of the core
- $N$ is the number of turns
- $A$ is the cross sectional area in square meters
- $l$ is the length in meters

air-core symbol ($\mu_r = 1$)

iron-core symbol ($\mu_r >> 1$)
The Unit of Inductance and Energy

- The henrys (H) is the unit of inductance.
  - One henrys of inductance means that one volt of voltage is induced due to a rate of change of one A/sec.

\[ v_L(t) = L \frac{di_L(t)}{dt} \]

\[ \frac{di_L(t)}{dt} = \frac{v_L(t)}{L} \rightarrow [i_L(t')]_{-\infty}^{t} = \int_{-\infty}^{t} \frac{v_L(t')}{L} \, dt' \]

\[ i_L(t) = i_L(t = 0) + \int_{0}^{t} \frac{v_L(t')}{L} \, dt' \]

\[ W_L(t) = \int_{0}^{t} p(t') \, dt' = \int_{0}^{t} v_L(t')i_L(t') \, dt' = \int_{0}^{t} L \frac{di_L(t)}{dt}i_L(t') \, dt' \]

\[ W_L(t) = \frac{1}{2} Li_L^2(t) \text{ Energy stored in an inductor (J)} \]

\[ L : \text{inductance in henrys} \quad i_L : \text{current in amperes} \]

Read the table 4.2 (Analogy between electric and fluid circuits) !!
Energy Accumulation & Dissipation

• **Energy accumulation** (left switch closed, right switch open)
  – The current flows through the inductor increasing up to $I_B$ and energy is stored.

• **Energy dissipation** (left switch open, right switch closed)
  – the energy accumulated on the inductor dissipates in the resistor.
Series and Parallel Inductances

• Series: Total \( L \) is the sum of individual \( L \)s:
  - \( L_T = L_1 + L_2 + \ldots \) etc.
• Current is the same through the series inductors.
  \[
  v = v_1 + v_2 + v_3 = L_1 \frac{di_1(t)}{dt} + L_2 \frac{di_2(t)}{dt} + L_3 \frac{di_3(t)}{dt} = (L_1 + L_2 + L_3) \frac{di(t)}{dt} = L_{EQ} \frac{di(t)}{dt}
  \]

• Parallel: Total \( L \) is less than the smallest individual value.
  - \( 1/L_T = 1/L_1 + 1/L_2 + \ldots \) etc.
• Voltage is the same across parallel inductors.
  \[
  i = i_1 + i_2 + i_3 = \frac{1}{L_1} \int_{-\infty}^{t} v_L(t')dt' + \frac{1}{L_2} \int_{-\infty}^{t} v_L(t')dt' + \frac{1}{L_3} \int_{-\infty}^{t} v_L(t')dt' = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right) \int_{-\infty}^{t} v_L(t')dt' = \frac{1}{L_{EQ}} \int_{-\infty}^{t} v_L(t')dt'
  \]

\( L_{EQ} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}} \)
Time-Dependent Signal Sources

- Consider sources that generate time-varying voltages and currents and, in particular, sinusoidal sources.

- One of the most important time-dependent signals is periodic signal.

\[ x(t) = x(t + nT) \quad n = 1, 2, 3, \ldots \text{ and } T \text{ is the period of } x(t) \]

- Several types of waveforms are provided by commercially available function (signal) generators.
Time-Dependent Signal Sources (cont.)

A generalized sinusoid is defined as
\[ x(t) = A \cos(\omega t + \phi) \] where \( A \) is the amplitude, \( \omega \) the radian frequency and \( \phi \) the phase (angle).

\[ x_1(t) = A \cos(\omega t) \quad \text{and} \quad x_2(t) = A \cos(\omega t + \phi) \]

where \( f \) is frequency \( = \frac{1}{T} \) (cycles or Hz)

\( \omega \) the radian frequency \( = \frac{d\theta}{dt} = 2\pi f \) (rad/sec)

\( \phi \) the phase (angle) \( = 2\pi \frac{\Delta t}{T} \) (rad) or \( = 360 \frac{\Delta t}{T} \) (deg)
Time-Dependent Signal Sources (cont.)

- The following specific values are used to compare one wave to another:
  - **Peak value**: Maximum value for currents or voltages. This applies to the positive or negative peak.
  - **Peak-to-peak**: Usually, but not always, double the peak value, as it measures distance between two amplitudes.
  - **Average value**: Arithmetic average of all values in one half-cycle (the full cycle average = 0).
  - **Root-Mean-Square (RMS) or Effective Value**: The amount of a sine wave of voltage or current that will produce the same power compared to the DC values.

\[
I = I_m \sin(\omega t = \theta)
\]

\[
\omega = \frac{d\theta}{dt} = \frac{\Delta \theta}{\Delta t} = \frac{\theta - 0}{t - 0} = \frac{\theta}{t}
\]

for constant angular velocity motion

The average value is 0.637 \times peak value.

The rms value is 0.707 \times peak value.

\[
P = I^2 R \quad \text{for DC cases}
\]

\[
I_{av} = \frac{1}{\pi} \int_0^\pi I d\theta = \frac{1}{\pi} \int_0^\pi I_m \sin \theta d\theta = \frac{2}{\pi} I_m
\]

\[
I_{rms} = \bar{I} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (I_m \sin \theta)^2 d\theta} = \frac{1}{\sqrt{2}} I_m, \quad P_{av} = \frac{1}{\pi} \int_0^{2\pi} I^2 R d\theta = R \frac{1}{\pi} \int_0^{2\pi} I^2 d\theta = I_{rms}^2 R
\]
**RMS vs. DC**

\[ V_{\text{rms}} = 120 \text{ V} \]

\[ \text{V}_{\text{rms}} \text{ is the effective value.} \]

The heating effect of these two sources is identical.

“Same power Dissipation” with rms values in AC
Phase Angle

- **Phase angle** ($\Theta$) is the **angular difference** between the same points on two different waveforms of the same frequency.
  - Two waveforms that have **peaks** and **zeros** at the same **time** are **in phase** and have a **phase angle** of $0^\circ$.
  - When one sine wave is at its **peak** while another is at **zero**, the two are $90^\circ$ **out of phase**.
  - When one sine wave has just the **opposite phase** of another, they are $180^\circ$ **out of phase**.

Two sine-wave voltages are $90^\circ$ out of phase.
The 60-Hz AC Power Line

• Almost all homes in the US are supplied alternating voltage between 115 and 125 V rms, at a frequency of 60 Hz.
  – Although the frequency of house wiring in North America is 60 Hz, many places outside N. America use a 50 Hz standard for house wiring.

• Residential wiring uses ac power instead of dc, because ac is more efficient in distribution from the generating station.

• House wiring in the US uses 3-wire, single-phase power.

• A value higher than 120 V would create more danger of fatal electric shock, but lower voltages would be less efficient in supplying power.
  – Higher voltage can supply electric power with less $I^2R$ loss.