Chapter 5  Flow Analysis Using Control Volume
MAIN TOPICS

- Conservation of Mass
- Newton’s Second Law – The Linear Momentum Equations
- The Angular Momentum Equations
- First Law of Thermodynamics – The Energy Equation
- Second Law of Thermodynamics – Irreversible Flow
Conservation of Mass – The Continuity Equation

- Basic Law for Conservation of Mass

\[
\frac{dM_{\text{system}}}{dt} = 0 \quad M_{\text{system}} = \int_{M(\text{system})} \text{dm} = \int_{V(\text{system})} \rho \text{d}V
\]

- For the system and a fixed, nondeforming control volume that are coincident at an instant of time, the Reynolds Transport Theorem leads to

\[
\frac{D}{Dt} \int_{\text{sys}} \rho dV \equiv \frac{\partial}{\partial t} \int_{\text{CV}} \rho dV + \int_{\text{CS}} \rho (\vec{V} \cdot \vec{n}) dA = 0
\]

- Time rate of change of the mass of the coincident system = Time rate of change of the mass of the content of the coincident control volume + Net rate of flow of mass through the control surface = 0
System and control volume at three different instances of time.

(a) System and control volume at time $t - \delta t$. (b) System and control volume at time $t$, coincident condition. (c) System and control volume at time $t + \delta t$. 
Conservation of Mass –
The Continuity Equation

For a fixed, nondeforming control volume, the control volume formulation of the conservation of mass: The continuity equation

\[
\frac{dM}{dt} = \sum m_{out} - \sum m_{in} \]

\[
\frac{\partial}{\partial t} \int_{CV} \rho \, dV + \int_{CS} \rho \vec{V} \cdot \hat{n} \, dA = 0
\]

Rate of increase of mass in CV

Net influx of mass

\[
\frac{\partial}{\partial t} \int_{CV} \rho \, dV = -\int_{CS} \rho \vec{V} \cdot \hat{n} \, dA
\]
Conservation of Mass –
The Continuity Equation 4/4

- **Incompressible Fluids**

\[
\rho \frac{\partial}{\partial t} \int_{CV} dV + \rho \int_{CS} \vec{V} \cdot \vec{n} dA = 0 \rightarrow \frac{\partial}{\partial t} \int_{CV} dV + \int_{CS} \vec{V} \cdot \vec{n} dA = 0
\]

- **For steady flow**

\[
\int_{CS} \rho \vec{V} \cdot \vec{n} dA = 0
\]

The mass flow rate into a control volume must be equal to the mass flow rate out of the control volume.
Other Definition

- **Mass flowrate through a section of control surface**

\[ \dot{m} = \rho Q = \int_A \rho \vec{V} \cdot \vec{n} dA = \sum \dot{m}_{\text{out}} - \sum \dot{m}_{\text{in}} \]

- **The average velocity**

\[ \overline{V} = \frac{\int_A \rho \vec{V} \cdot \vec{n} dA}{\rho A} \]
Fixed, Nondeforming Control Volume \( \frac{1}{2} \)

- **When the flow is steady**
  \[
  \frac{\partial}{\partial t} \int_{CV} \rho dV = 0 \quad \rightarrow \quad \sum \dot{m}_{\text{out}} = \sum \dot{m}_{\text{in}}
  \]

- **When the flow is steady and incompressible**
  \[
  \sum \dot{Q}_{\text{out}} = \sum \dot{Q}_{\text{in}}
  \]

- **When the flow is not steady**
  \[
  \frac{\partial}{\partial t} \int_{CV} \rho dV \neq 0 \quad \text{“+” : the mass of the contents of the control volume is increasing}
  \]
  \[
  \text{“-” : the mass of the contents of the control volume is decreasing.}
  \]
When the flow is uniformly distributed over the opening in the control surface (one dimensional flow)
\[ \dot{m} = \rho AV \]

When the flow is non-uniformly distributed over the opening in the control surface
\[ \dot{m} = \rho A \bar{V} \]
Example 5.1 Conservation of Mass – Steady, Incompressible Flow

- Seawater flows steadily through a simple conical-shaped nozzle at the end of a fire hose as illustrated in Figure E5.1. If the nozzle exit velocity must be at least 20 m/s, determine the minimum pumping capacity required in m³/s.

Figure E5.1
Example 5.1 Solution

The continuity equation

\[ \frac{\partial}{\partial t} \int_{CV} \rho \, dV + \int_{CS} \rho \, \vec{V} \cdot \vec{n} \, dA = 0 \]

\[ \int_{CS} \rho \, \vec{V} \cdot \vec{n} \, dA = (\rho VA)_2 + (-\rho VA)_1 = m_2 - m_1 = 0 \quad \text{or} \quad m_2 = m_1 \]

\[ \rho_1 Q_1 = \rho_2 Q_2 \]

With incompressible condition

\[ \rho_1 = \rho_2 \Rightarrow Q_1 = Q_2 = V_2 A_2 = ... = 0.025 \text{m}^3 / \text{s} \]
Example 5.2 Conservation of Mass – Steady, Compressible Flow

Air flows steadily between two sections in a long, straight portion of 4-in. inside diameter as indicated in Figure E5.2. The uniformly distributed temperature and pressure at each section are given. If the average air velocity (Non-uniform velocity distribution) at section (2) is 1000 ft/s, calculate the average air velocity at section (1).

\[ D_1 = D_2 = 4 \text{ in.} \]

\[ p_1 = 100 \text{ psia} \]
\[ T_1 = 540 \degree \text{R} \]
\[ p_2 = 18.4 \text{ psia} \]
\[ T_2 = 453 \degree \text{R} \]
\[ V_2 = 1000 \text{ ft/s} \]
Example 5.2 Solution

The continuity equation

\[ \frac{\partial}{\partial t} \int_{CV} \rho \, dV + \int_{CS} \rho \vec{V} \cdot \vec{n} \, dA = 0 \]

\[ \int_{CS} \rho \vec{V} \cdot \vec{n} \, dA = m_2 - m_1 = 0 \Rightarrow m_2 = m_1 \]

\[ \rho_1 A_1 \vec{V}_1 = \rho_2 A_2 \vec{V}_2 \]

Since \( A_1 = A_2 \)

\[ \vec{V}_1 = \frac{\rho_2}{\rho_1} \vec{V}_2 \]

\[ \vec{V}_1 = \frac{p_2 T_1}{p_1 T_2} \vec{V}_2 = \ldots 219 \text{ ft/s} \]

The ideal gas equation

\[ \rho = \frac{p}{RT} \]
Example 5.3 Conservation of Mass – Two Fluids

- Moist air (a mixture of dry air and water vapor) enters a dehumidifier at the rate of 22 slugs/hr. Liquid water drains out of the dehumidifier at a rate of 0.5 slugs/hr. Determine the mass flowrate of the dry air and the water vapor leaving the dehumidifier.
Example 5.3 Solution

The continuity equation

Steady flow

$$\frac{\partial}{\partial t} \int_{CV} \rho dV + \int_{CS} \rho \vec{V} \cdot \vec{n} dA = 0$$

$$\rho \int_{CS} \vec{V} \cdot \vec{n} dA = -\dot{m}_1 + \dot{m}_2 + \dot{m}_3 = 0$$

$$\dot{m}_2 = \dot{m}_1 - \dot{m}_3 = 22 \text{ slugs} / \text{hr} - 0.5 \text{ slugs} / \text{hr} = 21.5 \text{ slugs} / \text{hr}$$
Example 5.4 Conservation of Mass – Nonuniform Velocity Profiles

Incompressible, laminar water flow develops in a straight pipe having radius R as indicated in Figure E5.4. At section (1), the velocity profile is uniform; the velocity is equal to a constant value \( U \) and is parallel to the pipe axis everywhere. At section (2), the velocity profile is axisymmetric and parabolic, with zero velocity at the pipe wall and a maximum value of \( u_{\text{max}} \) at the centerline. How are \( U \) and \( u_{\text{max}} \) related? How are the average velocity at section (2), \( \overline{V}_2 \), and \( u_{\text{max}} \) related?
Example 5.4 Solution

The continuity equation

Steady flow

\[ \frac{\partial}{\partial t} \int_{CV} \rho \, dV + \int_{CS} \rho \, \vec{V} \cdot \vec{n} \, dA = 0 \]

\[ - \rho_1 A_1 U + \int_{A_2} \rho \, \vec{V} \cdot \vec{n} \, dA = 0 \]

With incompressible condition \( \rho_1 = \rho_2 \)

\[ - A_1 U + 2 \pi u_{\text{max}} \int_{0}^{R} \left[ 1 - \left( \frac{r}{R} \right)^2 \right] r \, dr = 0 \]

\[ u_{\text{max}} = 2U \]

\[ \overline{V}_2 = \frac{u_{\text{max}}}{2} \]
Example 5.5 Conservation of Mass – Unsteady Flow

A bathtub is being filled with water from a faucet. The rate of flow from the faucet is steady at 9 gal/min. The tub volume is approximated by a rectangular space as indicate Figure E5.5(a). Estimate the time rate of change of the depth of water in the tub, $\frac{\partial h}{\partial t}$, in in./min at any instant.
The continuity equation

\[ \frac{\partial}{\partial t} \int_{CV} \rho dV + \int_{CS} \rho \vec{V} \cdot \vec{n} dA = 0 \]

\[ \quad = \frac{\partial}{\partial t} \int_{\text{air volume}} \rho_{\text{air}} dV_{\text{air}} + \frac{\partial}{\partial t} \int_{\text{water volume}} \rho_{\text{water}} dV_{\text{water}} - \dot{m}_{\text{water}} + \dot{m}_{\text{air}} \]

For air \[ \frac{\partial}{\partial t} \int_{\text{air volume}} \rho_{\text{air}} dV_{\text{air}} + \dot{m}_{\text{air}} = 0 \]
Example 5.5 Solution

For water

\[
\frac{\partial}{\partial t} \int_{\text{water volume}} \rho_{\text{water}} \, dV_{\text{water}} = \dot{m}_{\text{water}}
\]

\[
\frac{\partial}{\partial t} \int_{\text{water volume}} \rho_{\text{water}} \, dV_{\text{water}} = \rho_{\text{water}} \left[ h(2\,\text{ft})(5\,\text{ft}) + (1.5\,\text{ft} - h)A_j \right]
\]

\[\Rightarrow \rho_{\text{water}} (10\,\text{ft}^2 - A_j) \frac{\partial h}{\partial t} = \dot{m}_{\text{water}}\]

\[
\frac{\partial h}{\partial t} = \frac{Q_{\text{water}}}{(10\,\text{ft}^2 - A_j)} = \frac{(9\,\text{gal/\text{min}})(12\,\text{in./ft})}{(7.48\,\text{gal/ft}^3)(10\,\text{ft}^2)}
\]

\[A_j \ll 10\,\text{ft}^2\]
Moving, Nondeforming Control Volume

- When a moving control volume is used, the fluid velocity relative to the moving control is an important variable.
  - \( \vec{W} \) is the relative fluid velocity seen by an observer moving with the control volume.
  - \( \vec{V}_{cv} \) is the control volume velocity as seen from a fixed coordinate system.
  - \( \vec{V} \) is the absolute fluid velocity seen by a stationary observer in a fixed coordinate system.

\[
\vec{W} = \vec{V} - \vec{V}_{cv} \quad \frac{DM_{sys}}{Dt} = \frac{\partial}{\partial t} \int_{cv} \rho dV + \int_{c.s.} \rho (\vec{W} \cdot \vec{n}) dA
\]

\[
\frac{\partial}{\partial t} \int_{cv} \rho dV + \int_{c.s.} \rho \vec{W} \cdot \vec{n} dA = 0
\]
Example 5.6 Conservation of Mass - Compressible Flow with a Moving Control Volume

An airplane moves forward at speed of 971 km/hr as shown in Figure E5.6 (a). The frontal intake area of the jet engine is 0.80 m² and the entering air density is 0.736 kg/m³. A stationary observer determines that relative to the earth, the jet engine exhaust gases move away from the engine with a speed of 1050 km/hr. The engine exhaust area is 0.558 m², and the exhaust gas density is 0.515 kg/m³. Estimate the mass flowrate of fuel into the engine in kg/hr.

Determine the mass flowrate of fuel into the engine in kg/hr
The continuity equation

$$\frac{\partial}{\partial t} \int_{CV} \rho dV + \int_{C.S.} \rho \vec{W} \cdot \vec{n} dA = 0$$

Assuming one-dimensional flow

$$- \dot{m}_{\text{fuel in}} - \rho_1 A_1 W_1 + \rho_2 A_2 W_2 = 0$$

$$\rightarrow \dot{m}_{\text{fuel in}} = \rho_2 A_2 W_2 - \rho_1 A_1 W_1$$

$$W_2 = V_2 - V_{\text{plane}} = 1050\text{km/hr} + 971\text{km/hr} = 201\text{km/hr}$$

$$\rightarrow \dot{m}_{\text{fuel in}} = (0.515\text{kg/m}^3)(0.558\text{m}^2)(2021\text{km/hr})(1000\text{m/km}) - \ldots = 9100\text{kg/hr}$$

The intake velocity, $W_1$, relative to the moving control volume. The exhaust velocity, $W_2$, also needs to be measured relative to the moving control volume.
Deforming Control Volume

- A deforming control volume involves changing volume size and control surface movement.
- The Reynolds transport theorem for a deforming control volume can be used for this case.

\[
\frac{DM_{sys}}{Dt} = \frac{\partial}{\partial t} \int_{CV} \rho dV + \int_{C.S.} \rho (\vec{W} \cdot \vec{n}) dA
\]

\[
\vec{V} = \vec{W} + \vec{V}_{CS}
\]

\( \vec{V}_{CS} \) is the velocity of the control surface as seen by a fixed observer. \( \vec{W} \) is the relative velocity referenced to the control surface.
Example 5.8 Conservation of Mass – Deforming Control Volume 1/2

A syringe is used to inoculate a cow. The plunger has a face area of 500 mm². If the liquid in the syringe is to be injected steadily at a rate of 300 cm³/min, at what speed should the plunger be advanced? The leakage rate past the plunger is 0.01 times the volume flowrate out of the needle.

Determine the speed of the plunger be advanced
Example 5.8 Solution

\[ A_1 \cong A_p \]

The continuity equation

\[ \frac{\partial}{\partial t} \int_{CV} \rho dV + \int_{c.s.} \rho \vec{W} \cdot \vec{n} dA = 0 \]

\[ \rightarrow \frac{\partial}{\partial t} \int_{CV} \rho dV + \dot{m}_2 + \rho Q_{\text{leak}} = 0 \]

Let

\[ - \frac{\partial \ell}{\partial t} = V_p \Rightarrow -\rho A_1 V_p + \dot{m}_2 + \rho Q_{\text{leak}} = 0 \]

\[ - \rho A_1 V_p + \rho Q_2 + \rho Q_{\text{leak}} = 0 \]

\[ V_p = \frac{Q_2 + Q_{\text{leak}}}{A_1} = \ldots = 660 \text{mm} / \text{min} \]
Newton’s second law for a system moving relative to an inertial coordinate system.

Time rate of change of the linear momentum of the system = Sum of external forces acting on the system

\[ \sum \vec{F}_{sys} = \sum \vec{F}_S + \sum \vec{F}_B = \frac{D}{Dt} \int_{sys} \vec{V} \rho d\vec{V} = \frac{D\vec{P}}{Dt}_{\text{system}} \]

\[ \vec{P}_{\text{system}} = \int_{M(\text{system})} \vec{V} \rho d\vec{V} = \int_{\mathcal{V}(\text{system})} \vec{V} \rho d\mathcal{V} \]
For the system and a fixed, nondeforming control volume that are coincident at an instant of time, the Reynolds Transport Theorem leads to

\[ \frac{D}{Dt} \int_{\text{sys}} \vec{V} \rho d\mathcal{V} \equiv \frac{\partial}{\partial t} \int_{\text{CV}} \vec{V} \rho d\mathcal{V} + \int_{\text{CS}} (\vec{n} \cdot \vec{V}) \rho \vec{V} dA \]

\[ \frac{D}{Dt} \int_{\text{sys}} \vec{V} \rho d\mathcal{V} \equiv \frac{\partial}{\partial t} \int_{\text{CV}} \vec{V} \rho d\mathcal{V} + \int_{\text{CS}} \vec{V} \rho (\vec{n} \cdot \vec{V}) dA \]

Time rate of change of the linear momentum of the coincident system = Time rate of change of the linear momentum of the content of the coincident control volume + Net rate of flow of linear momentum through the control surface

\( B = P \) and \( b = \vec{V} \)
When a control volume is coincident with a system at an instant of time, the force acting on the system and the force acting on the contents of the coincident control volume are instantaneously identical.

\[ \sum \vec{F}_{\text{sys}} = \sum \vec{F}_{\text{contents of the coincident control volume}} \]
For a fixed and nondeforming control volume, the control volume formulation of Newton’s second law

**Linear momentum equation**

\[
\frac{\partial}{\partial t} \int_{CV} \vec{V} \rho dV + \int_{CS} \vec{V} \rho (\vec{V} \cdot \vec{n}) dA = \sum \vec{F}
\]

Contents of the coincident control volume
Example 5.10 Linear Momentum – Change in Flow Direction

As shown in Figure E5.10 (a), a horizontal jet of water exits a nozzle with a uniform speed of $V_1 = 10$ ft/s, strike a vane, and is turned through an angle $\theta$. Determine the anchoring force needed to hold the vane stationary. Neglect gravity and viscous effects.

Determine the anchoring force needed to hold the vane stationary.
Example 5.10 Solution

The x and z direction components of linear momentum equation

\[ \frac{\partial}{\partial t} \int_{CV} u \rho dV + \int_{CS} u \rho \vec{V} \cdot \vec{n} dA = \sum F_x \]

\[ \vec{V} = u \vec{i} + w \vec{k} \]

\[ \frac{\partial}{\partial t} \int_{CV} w \rho dV + \int_{CS} w \rho \vec{V} \cdot \vec{n} dA = \sum F_z \]

\[ V_1 \rho (-V_1) A_1 + V_1 \cos \theta \rho (V_1) A_2 = F_{Ax} \]

\[ (0) \rho (-V_1) A_1 + V_1 \sin \theta \rho (V_1) A_2 = F_{Az} \]

\[ \rightarrow F_{Ax} = -\rho V_1^2 A_1 (1 - \cos \theta) = \ldots = -11.64 (1 - \cos \theta) \text{ lb} \]

\[ \rightarrow F_{Az} = \rho V_1^2 A_1 \sin \theta = \ldots = 11.64 \sin \theta \text{ lb} \]
Example 5.11 Linear Momentum – Weight, pressure, and Change in Speed

Determine the anchoring force required to hold in place a conical nozzle attached to the end of a laboratory sin faucet when the water flowrate is 0.6 liter/s. The nozzle mass is 0.1 kg. The nozzle inlet and exit diameters are 16 mm and 5 mm, respectively. The nozzle axis is vertical and the axial distance between section (1) and (2) is 30 mm. The pressure at section (1) is 464 kPa. to hold the vane stationary. Neglect gravity and viscous effects.
Example 5.11 Solution^{1/3}

- $D_1 = 16$ mm
- $h = 30$ mm
- $D_2 = 5$ mm

$F_A$ = anchoring force that holds nozzle in place
$W_n$ = weight of nozzle
$W_w$ = weight of water contained in the nozzle
$p_1$ = gage pressure at section (1)
$A_1$ = cross section area at section (1)
$p_2$ = gage pressure at section (2)
$A_2$ = cross section area at section (2)
$w_1$ = \( \tau \) direction velocity at control volume entrance
$w_2$ = \( \tau \) direction velocity at control volume exit
Example 5.11 Solution\textsuperscript{2/3}

The z direction component of linear moment equation

\[ \frac{\partial}{\partial t} \int_{CV} w \rho dV + \int_{CS} w \rho \vec{V} \cdot \vec{n} dA = F_A - W_n - p_1 A_1 - W_w + p_2 A_2 \]

\[ \vec{V} \cdot \vec{n} dA = \pm |w| dA \]

With the “+” used for flow out of the control volume and “-” used for flow in.

\[( - \dot{m}_1)(-w_1) + \dot{m}_2(-w_2) = -W_n - p_1 A_1 - W_w + p_2 A_2 \]

\[ F_A = \dot{m}(w_1 - w_2) + W_n + p_1 A_1 + W_w - p_2 A_2 \]

\[ \dot{m}_1 = \dot{m}_2 = \dot{m} \quad \dot{m}_1 = \dot{m}_2 = \dot{m} = \rho w_1 A_1 = \rho Q = ... = 0.599 \text{ kg/s} \]
Example 5.11 Solution

\[ w_1 = \frac{Q}{A_1} = \frac{Q}{\pi(D_1^2/4)} = ... = 2.98 \text{ m/s} \]

\[ w_2 = \frac{Q}{A_2} = \frac{Q}{\pi(D_2^2/4)} = ... = 30.6 \text{ m} \]

\[ W_n = m_n g = (0.1 \text{ kg})(9.81 \text{ m/s}^2) = 0.981 \text{ N} \]

\[ W_w = \rho V_w g = \rho \left[ \frac{1}{12} \pi h(D_1^2 + D_2^2 + D_1 D_2) \right] V_w g = ... = 0.0278 \text{ N} \]

\[ F_A = \dot{m}(w_1 - w_2) + W_n + p_1 A_1 + W_w - p_2 A_2 \]

\[ = (0.599 \text{ kg/s})(...) = ... = 77.8 \text{ N} \]
Water flows through a horizontal, 180° pipe bend. The flow cross-section area is constant at a value of 0.1 ft\(^2\) through the bend. The magnitude of the flow velocity everywhere in the bend is axial and 50 ft/s. The absolute pressure at the entrance and exit of the bend are 30 psia and 24 psia, respectively. Calculate the horizontal (x and y) components of the anchoring force required to hold the bend in place.
Example 5.12 Solution

The x direction component of linear moment equation

\[ \frac{\partial}{\partial t} \int_{CV} u \rho dV + \int_{CS} u \rho \vec{V} \cdot \vec{n} dA = F_{Ax} \]

\[ \vec{F}_S = \int_A - \vec{n} p dA \]

At section (1) and (2), the flow is in the y direction and therefore \( u = 0 \) at both sections.

\[ F_{Ax} = 0 \]

The y direction component of linear moment equation

\[ \frac{\partial}{\partial t} \int_{CV} v \rho dV + \int_{CS} v \rho \vec{V} \cdot \vec{n} dA = F_{Ay} + p_1 A_1 + p_2 A_2 \]
Example 5.12 Solution

For one-dimensional flow

\((+v_1)(-\dot{m}_1) + (-v_2)(+\dot{m}_2) = F_{Ay} + p_1A_1 + p_2A_2\)

\[- \dot{m}(v_1 + v_2) = F_{Ay} + p_1A_1 + p_2A_2\]

\[F_{Ay} = - \dot{m}(v_1 + v_2) - p_1A_1 - p_2A_2 = ... = -1324 \text{lb}\]

\[\dot{m}_1 = \dot{m}_2 = \dot{m} = \rho v_1 A_1 = ... = 9.70 \text{slugs} / \text{s}\]
Example 5.13 Linear Momentum – Weight, pressure, and Change in Speed

Air flows steadily between two cross sections in a long, straight portion of 4-in. inside diameter pipe as indicated in Figure E5.13, where the uniformly distributed temperature and pressure at each cross section are given. If the average air velocity at section (2) is 1000 ft/s, we found in Example 5.2 that the average air velocity at section (1) must be 219 ft/s. Assuming uniform velocity distributions at sections (1) and (2), determine the frictional force exerted by the pipe wall on the air flow between sections (1) and (2).
Example 5.13 Solution$^{1/2}$

The axial component of linear moment equation

\[
\frac{\partial}{\partial t}\int_{CV} u\rho dV + \int_{CS} u\rho \vec{V} \cdot \vec{n} dA = -R_x + p_1A_1 - p_2A_2
\]

\[
(+u_1)(-\dot{m}_1) + (+u_2)(+\dot{m}_2) = -R_x + p_1A_1 - p_2A_2
\]

\[
\dot{m}(u_2 - u_1) = -R_x + A_2(p_1 - p_2)
\]

\[
R_x = A_2(p_1 - p_2) - \dot{m}(u_2 - u_1)
\]

\[
m_1 = \dot{m}_2 = \dot{m} = \left(\frac{p_2}{RT_2}\right)\left(\frac{\pi D_2^2}{4}\right)u_2 = ... = 0.297 \text{ slugs / s}
\]
Example 5.13 Solution

\[ R_x = A_2(p_1 - p_2) - \dot{m}(u_2 - u_1) \]

\[ R_x = A_2(p_1 - p_2) - \dot{m}(u_2 - u_1) = \ldots = 793 \text{ lb} \]

\[ \rho_2 = \frac{p_2}{RT_2} \]

\[ A_2 = \frac{\pi D_2^2}{4} \]
If the flow of Example 5.4 is vertically upward, develop an expression for the fluid pressure drop that occurs between sections (1) and (2).
Example 5.14 Solution

The axial component of linear moment equation

\[
\frac{\partial}{\partial t} \int_{CV} w \rho dV + \int_{CS} w \rho \vec{V} \cdot \hat{n} dA = p_1 A_1 - R_z - W - p_2 A_2
\]

\[\implies (+w_1)(-\dot{m}_1) + \int_{CS} (+w_2) \rho (+w_2 dA_2) = p_1 A_1 - R_z - W - p_2 A_2\]

\[w_2 = 2w_1 \left[1 - \left(\frac{r}{R}\right)^2\right]\]

\[\int_{CS} (+w_2) \rho (+w_2 dA_2) = \rho \int_0^R w_2^2 2\pi r dr = 4\pi \rho w_1^2 \frac{R^2}{3}\]

\[- w_1^2 \rho \pi R^2 + \frac{4}{3} w_1^2 \rho \pi R = p_1 A_1 - R_z - W - p_2 A_2\]

\[\implies p_1 - p_2 = \frac{\rho w_1^2}{3} + \frac{R_z}{A_1} + \frac{W}{A_1}\]
A static thrust as sketched in Figure E5.15 is to be designed for testing a jet engine. The following conditions are known for a typical test: Intake air velocity = 200 m/s; exhaust gas velocity = 500 m/s; intake cross-section area = 1 m²; intake static pressure = -22.5 kPa = 78.5 kPa (abs); intake static temperature = 268K; exhaust static pressure = 0 kPa = 101 kPa (abs). Estimate the normal thrust for which to design.
Example 5.15 Solution

The x direction component of linear moment equation

\[
\frac{\partial}{\partial t} \int_{CV} u \rho dV + \int_{CS} u \rho \vec{V} \cdot \vec{n} dA = p_1 A_1 + F_{th} - p_2 A_2 - p_{atm} (A_1 - A_2)
\]

\[
\Rightarrow (+u_1)(-\dot{m}_1) + (+u_2)(+\dot{m}_2) = (p_1 - p_{atm})A_1 - (p_2 - p_{atm})A_2 + F_{th}
\]

\[
\dot{m} = \dot{m}_1 = \rho_1 A_1 u_1 = \dot{m}_2 = \rho_2 A_2 u_2
\]

\[
\Rightarrow \dot{m}(u_2 - u_1) = p_1 A_1 - p_2 A_2 + F_{th}
\]

\[
F_{th} = -\rho_1 A_1 + \rho_2 A_2 + \dot{m}(u_2 - u_1) = ... = 83700 \text{N}
\]

\[
\rho_1 = \frac{p_1}{RT_1}
\]

\[
\dot{m} = \rho_1 A_1 u_1 = ... = 204 \text{kg/s}
\]
Example 5.16 Linear Momentum – Nomuniform Pressure

- A sluice gate across a channel of width b is shown in the closed and open position in Figure E5.16(a) and (b). Is the anchoring force required to hold the gate in place larger when the gate is closed or when it is open?
Example 5.16 Solution

When the gate is closed, the horizontal forces acting on the contents of the control volume are identified in Figure E5.16 (c).

\[
\int_{CS} u \rho \vec{V} \cdot \vec{n} dA = \frac{1}{2} \gamma H^2 b - R_x \Rightarrow R_x = \frac{1}{2} \gamma H^2 b
\]

When the gate is open, the horizontal forces acting on the contents of the control volume are identified in Figure E5.16 (d).

\[
\int_{CS} u \rho \vec{V} \cdot \vec{n} dA = \frac{1}{2} \gamma H^2 b - R_x - \frac{1}{2} \gamma h^2 b - F_f
\]

\[- \rho u_1 \gamma H^2 b + \rho u_2 \gamma h^2 b = \frac{1}{2} \gamma H^2 b - R_x - \frac{1}{2} \gamma h^2 b - F_f\]

For \( H \gg h \) and \( u_1 \ll u_2 \) \( \Rightarrow R_x = \frac{1}{2} \gamma H^2 b - \frac{1}{2} \gamma h^2 b - F_f - \rho u_2 \gamma h^2 b \)
Chapter 4: Reynolds transport equation for a control volume moving with constant velocity is

\[
\frac{DB_{sys}}{Dt} = \frac{\partial}{\partial t} \int_{CV} \rho b d\mathbf{V} + \int_{CS} \rho b \mathbf{W} \cdot \mathbf{n} dA
\]

\[
\frac{D}{Dt} \int_{sys} \mathbf{V} \rho d\mathbf{V} \equiv \frac{\partial}{\partial t} \int_{CV} \mathbf{V} \rho d\mathbf{V} + \int_{CS} \mathbf{V} \rho \mathbf{W} \cdot \mathbf{n} dA = \sum \mathbf{F}
\]

\[
\mathbf{V} = \mathbf{W} + \mathbf{V}_{CV}
\]

\[
\frac{\partial}{\partial t} \int_{CV} (\mathbf{W} + \mathbf{V}_{CV}) \rho d\mathbf{V} + \int_{CS} (\mathbf{W} + \mathbf{V}_{CV}) \rho \mathbf{W} \cdot \mathbf{n} dA = \sum \mathbf{F}
\]
Moving, Nondeforming Control Volume

For a constant control volume velocity, $V_{cv}$, and **steady flow** in the control volume reference frame

\[
\frac{\partial}{\partial t} \int_{CV} \left( \vec{W} + \vec{V}_{cv} \right) \rho dV = 0
\]

For steady flow, continuity equation

\[
\int_{CS} \left( \vec{W} + \vec{V}_{cv} \right) \rho \vec{W} \cdot \vec{n} dA = \int_{CS} \vec{W} \rho \vec{W} \cdot \vec{n} dA + \vec{V}_{cv} \int_{CS} \rho \vec{W} \cdot \vec{n} dA
\]

\[
\int_{C.S} \rho \vec{W} \cdot \vec{n} dA = 0
\]

\[
\frac{DM_{sys}}{Dt} = \frac{\partial}{\partial t} \int_{CV} \rho dV + \int_{C.S} \rho \vec{W} \cdot \vec{n} dA = 0
\]
For an inertial, moving, nondeforming control volume, the linear momentum equation of steady flow is given by:

\[ \int_{CS} \vec{W} \rho (\vec{W} \cdot \vec{n}) dA = \sum \vec{F} \]

Contents of the coincident control volume
The sum of all forces (surface and body forces) acting on a Non-accelerating control volume is equal to the sum of the rate of change of momentum inside the control volume and the net rate of flux of momentum out through the control surface.

\[
\sum \vec{F}_{\text{contents of the coincident control volume}} = \sum \vec{F}_S + \sum \vec{F}_B
\]

\[
= \frac{\partial}{\partial t} \int_{CV} \vec{V} \rho dV + \int_{CS} \vec{V} \rho (\vec{V} \cdot \hat{n}) dA
\]

\[
\vec{F}_B = \int \vec{B} dm = \int_{CV} \vec{B} \rho dV
\]

\[
\vec{F}_S = \int_{A} - \vec{n} p dA
\]

Where the velocities are measured Relative to the control volume.
Example 5.17 Linear Momentum - Moving Control Volume

A vane on wheels move with a constant velocity \( V_0 \) when a stream of water having a nozzle exit velocity of \( V_1 \) is turned 45° by the vane as indicated in Figure E5.17(a). Note that this is the same moving vane considered in Section 4.4.6 earlier. Determine the magnitude and direction of the force, \( F \), exerted by the stream of water on the vane surface. The speed of the water jet leaving the nozzle is 100 ft/s, and the vane is moving to the right with a constant speed of 20 ft/s.
Example 5.17 Linear Momentum - Moving Control Volume 2/2

F I G U R E E5.17

(a) Nozzle $A_1 = 0.006$ ft$^2$, $V_1$, $V_0$, $45^\circ$ Moving vane

(b) Moving control volume $V_{CV} = V_0$, $1$ ft Moving vane

(c) $R_z$, $R_x$, $W_w$, $x$, $z$
Example 5.17 Solution^{1/2}

The x direction component of linear moment equation

\[ \int_{CS} W_x \rho \vec{W} \cdot \vec{n} dA = -R_x \]

\[ \Rightarrow (+W_1)(-\hat{m}_1) + (+W_2 \cos 45^\circ)(+\hat{m}_2) = -R_x \]

\[ \hat{m}_1 = \rho_1 W_1 A_1 \quad \hat{m}_2 = \rho_2 W_2 A_2 \]

The z direction component of linear moment equation

\[ \int_{CS} W_z \rho \vec{W} \cdot \vec{n} dA = R_z - W_w \]

\[ \Rightarrow (+W_2 \sin 45^\circ)(+\hat{m}_2) = R_z - W_w \]

\[ \hat{m}_1 = \rho_1 W_1 A_1 = \hat{m}_2 = \rho_2 W_2 A_2 \]

\[ W_1 = W_2 = V_1 - V_0 = \ldots \]
Example 5.17 Solution

\[ R_x = \rho W_1^2 A_1 (1 - \cos 45^\circ) = \ldots = 21.8 \text{lb} \]

\[ R_z = \rho W_1^2 A_1 \sin 45^\circ + W_w = \ldots = 53 \text{lb} \]

\[ W_w = \rho g A_1 \ell \]

\[ R = \sqrt{R_x^2 + R_z^2} = \ldots = 57.3 \text{lb} \]

\[ \alpha = \tan^{-1} \frac{R_z}{R_x} \]
The first law of thermodynamics for a system is:

\[
\frac{D}{Dt} \int_{sys} e \rho dV = \left( \sum \dot{Q}_{in} - \sum \dot{Q}_{out} \right)_{sys} + \left( \sum \dot{W}_{in} - \sum \dot{W}_{out} \right)_{sys} = \left( \dot{Q}_{net/in} + \dot{W}_{net/in} \right)_{sys}
\]

or

\[
\frac{D}{Dt} \int_{sys} e \rho dV = \left( \dot{Q}_{net/in} + \dot{W}_{net/in} \right)_{sys}
\]

where \( e = \hat{u} + \frac{V^2}{2} + gz \)

Total stored energy per unit mass for each particle in the system

The net rate of work transfer into the system

The net rate of heat transfer into the system

“+” going into system

“-” coming out
First Law of Thermodynamics –
The Energy Equation$^{2/4}$

- For the system and the contents of the coincident control volume that is fixed and nondeforming -- Reynolds Transport Theorem leads to

$$\frac{D}{Dt} \int_{sys} e \rho dV = \frac{\partial}{\partial t} \int_{CV} e \rho dV + \int_{c.s.} e \rho \vec{V} \cdot \vec{n} dA$$

- Time rate of increase of the total stored energy of the system = Net time rate of increase of the total stored energy of the contents of the control volume + The net rate of flow of the total stored energy out of the control volume through the control surface
First Law of Thermodynamics – The Energy Equation

- For the control volume that is coincident with the system at an instant of time.

\[
(\dot{Q}_{\text{net in}} + \dot{W}_{\text{net in}})_{\text{sys}} = (\dot{Q}_{\text{net in}} + \dot{W}_{\text{net in}})_{\text{coincident control volume}}
\]

- The control volume formula for the first law of thermodynamics:

\[
\frac{\partial}{\partial t} \int_{cv} \rho \varepsilon dV + \int_{CS} \rho \vec{V} \cdot \vec{n} dA = (\dot{Q}_{\text{net in}} + \dot{W}_{\text{net in}})_{CV}
\]
Rate of Work done by CV

\[ \dot{W} = \dot{W}_{\text{shaft}} + \dot{W}_{\text{normal}} + \dot{W}_{\text{shear}} + \dot{W}_{\text{other}} \]

- **Shaft work** \( \dot{W}_{\text{shaft}} \): the rate of work transferred into through the CS by the shaft work (negative for work transferred out, positive for work input required)

- **Work done by normal stresses on the CS:**
  \[ \dot{W}_{\text{normal}} = \delta F_{\text{normal}} \cdot \vec{V} = -\int_{CS} p\vec{V} \cdot \vec{n} dA \]

- **Work done by shear stresses on the CS:**
  \[ \dot{W}_{\text{shear}} = +\int_{CS} \tau\vec{V} \cdot \vec{n} dA \text{ Negligibly small on a control surface} \]

- **Other work**

\[ \frac{\partial}{\partial t} \int_{cv} e \rho dV + \int_{CS} e \rho \vec{V} \cdot \vec{n} dA = \dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft net in}} - \int_{CS} p\vec{V} \cdot \vec{n} dA \]
First Law of Thermodynamics –
The Energy Equation

\[
\frac{\partial}{\partial t} \int_{CV} e \rho dV + \int_{CS} e \rho \vec{V} \cdot \vec{n} dA = \dot{Q}_{\text{net in}} + \dot{W}_{\text{Shaft net in}} - \int_{CS} p \vec{V} \cdot \vec{n} dA
\]

Energy equation

\[
\frac{\partial}{\partial t} \int_{CV} e \rho dV + \int_{CS} \left( \hat{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right) \rho \vec{V} \cdot \vec{n} dA = \dot{Q}_{\text{net/in}} + \dot{W}_{\text{Shaft/in}}
\]

\[
e = \hat{u} + \frac{V^2}{2} + gz
\]
Application of Energy Equation \(1/2\)

When the flow is steady, \(\frac{\partial}{\partial t} \int_{CV} \rho \epsilon dV = 0\)

The integral of

\[
\int_{CS} \left[ \hat{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right] \rho \vec{V} \cdot \vec{n} dA
\]

Uniformly distribution

\[
\int_{CS} \left[ \hat{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right] \rho \vec{V} \cdot \vec{n} dA = \sum_{\text{out}} \left( \hat{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right) \dot{m} - \sum_{\text{in}} \left( \hat{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right) \dot{m}
\]

Only one stream entering and leaving

\[
\int_{CS} \left[ \hat{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right] \rho \vec{V} \cdot \vec{n} dA = \left( \hat{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right)_{\text{out}} \dot{m}_{\text{out}} - \left( \hat{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right)_{\text{in}} \dot{m}_{\text{in}}
\]
If shaft work is involved, the one-dimensional energy equation for steady-in-the-mean flow is given by:

\[
\dot{m}\left[\hat{h}_{\text{out}} - \hat{h}_{\text{in}} + \left(\frac{p}{\rho}\right)_{\text{out}} - \left(\frac{p}{\rho}\right)_{\text{in}} + \frac{V_{\text{out}}^2 - V_{\text{in}}^2}{2} + g(z_{\text{out}} - z_{\text{in}})\right] = \dot{Q}_{\text{net/in}} + \dot{W}_{\text{shaft net/in}}
\]

Enthalpy: \( \hat{h} = \hat{u} + \frac{p}{\rho} \)  

The energy equation is written in terms of enthalpy.
Example 5.20 Energy – Pump Power 1/2

A pump delivers water at a steady rate of 300 gal/min as shown in Figure E5.20. Just upstream of the pump [section (1)] where the pipe diameter is 3.5 in., the pressure is 18 psi. Just downstream of the pump [section (2)] where the pipe diameter is 1 in., the pressure is 60 psi. The change in water elevation across the pump is zero. The rise in internal energy of water, \( u_2 - u_1 \), associated with a temperature rise across the pump is 3000 ft·lb/slug. If the pumping process is considered to be adiabatic, determine the power (hp) required by the pump.
**Example 5.20 Energy – Pump Power**

Control volume

\[ D_1 = 3.5 \text{ in.} \]

\[ p_1 = 18 \text{ psi} \]

\[ d_2 = 1 \text{ in.} \]

\[ Q = 300 \text{ gal/min.} \]

\[ p_2 = 60 \text{ psi} \]

\[ \ddot{u}_2 - \ddot{u}_1 = 3000 \text{ ft} \cdot \text{lb/slug} \]

\[ \dot{W}_{\text{shaft}} = ? \]
Example 5.20 Solution

One-dimensional energy equation for steady-in-the-mean flow

\[ \dot{m} \left[ \hat{u}_2 - \hat{u}_1 + \left( \frac{p}{\rho} \right)_2 - \left( \frac{p}{\rho} \right)_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right] = 0 \text{(Adiabatic flow)} \]

\[ \dot{Q}_{\text{net/in}} + \dot{W}_{\text{shaft net/in}} \]

\[ \dot{m} = \rho Q = \frac{(1.94 \text{slug} / \text{ft}^3)(300 \text{gal} / \text{min})}{(7.48 \text{gal} / \text{ft}^3)(60 \text{s} / \text{min})} = 1.30 \text{slugs / s} \]

\[ V = \frac{Q}{\pi D^2/4} \quad V_1 = \frac{Q}{A_1} = ... = 10.0 \text{ft / s} \quad V_2 = \frac{Q}{A_2} = ... = 123 \text{ft / s} \]

\[ \dot{W}_{\text{shaft net in}} = (1.30 \text{slugs / s})[... \ldots] = 32.3 \text{hp} \]
Example 5.21 Energy – Turbine Power per Unit Mass of Flow

Steam enters a turbine with a velocity of 30 m/s and enthalpy, $h_1$, of 3348 kJ/kg. The steam leaves the turbine as a mixture of vapor and liquid having a velocity of 60 m/s and an enthalpy of 2550 kJ/kg. If the flow through the turbine is adiabatic and changes in elevation are negligible, determine the work output involved per unit mass of steam through-flow.
Example 5.21 Solution

The energy equation in terms of enthalpy.

\[ \dot{m} \left[ \hat{h}_2 - \hat{h}_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right] = \dot{Q}_{\text{net/in}} + \dot{W}_{\text{shaft net/in}} \]

\[ \dot{W}_{\text{shaft net in}} = \frac{\dot{W}_{\text{shaft net in}}}{\dot{m}} = \hat{h}_2 - \hat{h}_1 + \frac{V_2^2 - V_1^2}{2} \]

\[ \dot{W}_{\text{shaft net out}} = -\dot{W}_{\text{shaft net in}} \]

\[ \dot{W}_{\text{shaft net out}} = \hat{h}_1 - \hat{h}_2 + \frac{V_1^2 - V_2^2}{2} = \ldots = 797 \text{kJ/kg} \]
Example 5.22 Energy – Temperature Change

- A 500-ft waterfall involves steady flow from one large body of water to another. Determine the temperature change associated with this flow.
Example 5.22 Solution

The temperature change is related to the change of internal energy of the water

\[ T_2 - T_1 = \frac{\hat{u}_2 - \hat{u}_1}{\tilde{c}} \]

where \( \tilde{c} = 1 \text{ Btu}/(\text{lbm} \cdot \text{°R}) \) is the specific heat of water

One-dimensional energy equation for steady-in-the-mean flow without shaft work

\[
\dot{m} \left[ \hat{u}_2 - \hat{u}_1 + \left( \frac{p_2}{\rho} \right) - \left( \frac{p_1}{\rho} \right) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right] = \dot{Q}_{\text{net in}} = 0 \text{(Adiabatic flow)}
\]

\[
T_2 - T_1 = \frac{g(z_2 - z_1)}{\tilde{c}} = \frac{(32.2 \text{ ft} / \text{s}^2)(500 \text{ ft})}{[778 \text{ ft} \cdot \text{lb} / (\text{lbm} \cdot \text{°R})][32.2(\text{lbm} \cdot \text{ft})/(\text{lb} \cdot \text{s}^2)]} = 0.643 \text{°R}
\]
Energy Equation vs. Bernoulli Equation

For steady, incompressible flow... One-dimensional energy equation

\[
\dot{m} \left[ \dot{u}_{out} - \dot{u}_{in} + \left( \frac{p_{out}}{\rho} \right) - \left( \frac{p_{in}}{\rho} \right) + \frac{V_{out}^2 - V_{in}^2}{2} + g(z_{out} - z_{in}) \right] = \dot{Q}_{net \ in}
\]

\[
\frac{p_{out}}{\rho} + \frac{V_{out}^2}{2} + gz_{out} = \frac{p_{in}}{\rho} + \frac{V_{in}^2}{2} + gz_{in} - \left( \dot{u}_{out} - \dot{u}_{in} - q_{net \ in} \right)
\]

where \( q_{net \ in} = \dot{Q}_{net \ in} / \dot{m} \)

For steady, incompressible, \textit{frictionless flow}...

\[
p_{out} + \frac{\rho V_{out}^2}{2} + \gamma Z_{out} = p_{in} + \frac{\rho V_{in}^2}{2} + \gamma Z_{in}
\]

\[
\dot{u}_{out} - \dot{u}_{in} - q_{net \ in} = 0
\]

\textit{Bernoulli equation}

\textit{Frictionless flow}...
For steady, incompressible, \textit{frictional flow}...

\( \hat{u}_{\text{out}} - \hat{u}_{\text{in}} - q_{\text{net in}} > 0 \) \textbf{Frictional flow}...

Defining “useful or available \textit{energy}”… \( \frac{p}{\rho} + \frac{V^2}{2} + gz \)

Defining “\textit{loss} of useful or available energy”… \( \hat{u}_{\text{out}} - \hat{u}_{\text{in}} - q_{\text{net in}} = \text{loss} \)

\( \frac{p_{\text{out}}}{\rho} + \frac{V_{\text{out}}^2}{2} + gz_{\text{out}} = \frac{p_{\text{in}}}{\rho} + \frac{V_{\text{in}}^2}{2} + gz_{\text{in}} - \text{loss} \)
For steady, incompressible flow with friction and shaft work...

\[
\dot{m}\left[\hat{u}_{out} - \hat{u}_{in} + \left(\frac{p_{out}}{\rho}\right) - \left(\frac{p_{in}}{\rho}\right) + \frac{V_{out}^2 - V_{in}^2}{2} + g(z_{out} - z_{in})\right] = \dot{Q}_{net \ in} + \dot{W}_{shaft \ net \ in}
\]

\[
\dot{m} \mid \frac{\rho}{p} \mid \frac{V_{out}^2}{2} + gz_{out} \mid \frac{\rho}{p} \mid \frac{V_{in}^2}{2} + gz_{in} + w_{shaft \ net \ in} - (\hat{u}_{out} - \hat{u}_{in} - q_{net \ in})
\]

\[
\dot{m} \mid \frac{\rho}{p} \mid \frac{V_{out}^2}{2} + gz_{out} \mid \frac{\rho}{p} \mid \frac{V_{in}^2}{2} + gz_{in} + w_{shaft \ net \ in} - \text{loss}
\]

\[
\dot{g} \mid \frac{p_{out}}{\gamma} \mid \frac{V_{out}^2}{2g} + z_{out} = \frac{p_{in}}{\gamma} \mid \frac{V_{in}^2}{2g} + z_{in} + h_s - h_L
\]

Shaft head \( h_s = \frac{w_{shaft\ net/in}}{g} \) = \( \dot{W}_{shaft\ net/in} \) = \( \dot{W}_{shaft\ net/in} \)

Head loss \( h_L = \frac{\text{loss}}{g} \)
Energy Equation & Bernoullii Equation

\[
\frac{p_{\text{out}}}{\gamma} + \frac{V_{\text{out}}^2}{2g} + z_{\text{out}} = \frac{p_{\text{in}}}{\gamma} + \frac{V_{\text{in}}^2}{2g} + z_{\text{in}} + h_s - h_L
\]

- For turbine: \( h_s = -h_T \) \((h_T > 0)\) \( h_T \) is turbine head
- For pump: \( h_s = h_p \) \( h_p \) is pump head
- The actual head drop across the turbine:
  \( h_T = -\left( h_s + h_L \right)_T \)
- The actual head drop across the pump:
  \( h_p = \left( h_s - h_L \right)_p \)
Example 5.23 Energy – Effect of Loss of Available Energy

- Compare the volume flowrates associated with two different vent configurations, a cylindrical hole in the wall having a diameter of 120 mm and the same diameter cylindrical hole in the wall but with a well-rounded entrance (see Figure E5.23a). The room pressure is held constant at 0.1 kPa above atmospheric pressure. Both vents exhaust into the atmosphere. As discussed in Section 8.4.2, the loss in available energy associated with flow through the cylindrical bent from the room to the vent exit is $0.5V_2^2/2$ where $V_2$ is the uniformly distributed exit velocity of air. The loss in available energy associated with flow through the rounded entrance vent from the room to the vent exit is $0.05V_2^2/2$, where $V_2$ is the uniformly distributed exit velocity of air.
For steady, incompressible flow with friction, the energy equation

\[ V_1 = 0 \quad \text{No elevation change} \]

\[ \frac{p_2}{\rho} + \frac{V_2^2}{2} + g z_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2} + g z_1 - \rho \Delta s_2 \]

\[ V_2 = \sqrt{2 \left[ \left( \frac{p_1 - p_2}{\rho} \right) \rho \Delta s_2 \right]} \quad \rho \Delta s_2 = K_L \frac{V_2^2}{2} \]

\[ V_2 = \sqrt{\frac{p_1 - p_2}{\rho \left( 1 + K_L / 2 \right)}} \]

\[ Q = A_2 V_2 = \frac{\pi D_2^2}{4} \sqrt{\frac{p_1 - p_2}{\rho \left( 1 + K_L / 2 \right)}} \]
Example 5.24 Energy – Fan Work and Efficiency

An axial-flow ventilating fan driven by a motor that delivers 0.4 kW of power to the fan blades produces a 0.6-m-diameter axial stream of air having a speed of 12 m/s. The flow upstream of the fan involves negligible speed. Determine how much of the work to the air actually produces a useful effects, that is, a rise in available energy and estimate the fluid mechanical efficiency of this fan.
Example 5.24 Solution

For steady, incompressible flow with friction and shaft work...

\[
W_{\text{shaft net in}} - \text{loss} = \left( \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 \right) - \left( \frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 \right)
\]

\( p_1 = p_2 = \text{atmospheric pressure}, \; V_1 = 0, \; \text{no elevation change} \)

\[
W_{\text{shaft net in}} - \text{loss} = \frac{V_2^2}{2} = 72.0 \text{N} \cdot \text{m} / \text{kg}
\]

Efficiency \( \eta = \frac{W_{\text{shaft net in}}}{W_{\text{shaft net in}} - \text{loss}} \)

\[
W_{\text{shaft net in}} = \frac{\dot{W}_{\text{shaft net in}}}{\dot{m}} = \frac{\dot{W}_{\text{shaft net in}}}{\rho AV} = 95.8 \text{N} \cdot \text{m} / \text{kg}
\]
Example 5.25 Energy – Head Loss and Power Loss

The pump shown in Figure E5.25 adds **10 horsepower** to the water as it pumps water from the lower lake to the upper lake. The elevation difference between the lake surfaces is 30 ft and the head loss is 15 ft. Determine the flowrate and power loss associated with this flow.
Example 5.25 Solution

The energy equation

\[ \frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_s - h_L \]

\[ p_A = p_B = 0 \quad V_A = V_B = 0 \]

The pump head

\[ h_s = h_L + z_A - z_B = \frac{\dot{W}_{shaft\ net/in}}{\gamma Q} = 88.1/ Q = (15 + 30 - 0) \text{ft} \]

Power loss

\[ \dot{W}_{loss} = \gamma Q h_L = \ldots \]
Application of Energy Equation to Nonuniform Flows

If the velocity profile at any section where flow crosses the control surface is not uniform...

$$\int_{C.S.} \frac{V^2}{2} \rho \vec{V} \cdot \vec{n} dA$$

For one stream of fluid entering and leaving the control volume...

$$\int_{CS} \frac{V^2}{2} \rho \vec{V} \cdot \vec{n} dA = m \left( \frac{\alpha_{out} \vec{V}_{out}^2}{2} - \frac{\alpha_{in} \vec{V}_{in}^2}{2} \right)$$

\(\alpha\) is the kinetic energy coefficient and \(V\) is the instantaneous velocity.

For uniform velocity profile, what is the coefficient?

$$\alpha = \frac{\int \frac{V^2}{2} \rho \vec{V} \cdot \vec{n} dA}{m \left( \frac{\vec{V}^2}{2} \right)} = \frac{\int \rho V^3 dA}{\int m \vec{V}^2} \geq 1$$
Application of Energy Equation to Nonuniform Flows

For nonuniform velocity profile:

\[ \frac{p_{\text{out}}}{\rho} + \frac{\alpha_{\text{out}} \bar{V}^2_{\text{out}}}{2} + g\bar{z}_{\text{out}} = \frac{p_{\text{in}}}{\rho} + \frac{\alpha_{\text{in}} \bar{V}^2_{\text{in}}}{2} + g\bar{z}_{\text{in}} + \rho w_{\text{shaft net in}} - \text{loss} \]

× \rho

\[ p_{\text{out}} + \frac{\rho \alpha_{\text{out}} \bar{V}^2_{\text{out}}}{2} + \gamma \bar{z}_{\text{out}} = p_{\text{in}} + \frac{\rho \alpha_{\text{in}} \bar{V}^2_{\text{in}}}{2} + \gamma \bar{z}_{\text{in}} + \rho w_{\text{shaft net in}} - \rho (\text{loss}) \]

÷ g

\[ \frac{p_{\text{out}}}{\gamma} + \frac{\alpha_{\text{out}} \bar{V}^2_{\text{out}}}{2g} + \frac{z_{\text{out}}}{\gamma} = \frac{p_{\text{in}}}{\gamma} + \frac{\alpha_{\text{in}} \bar{V}^2_{\text{in}}}{2g} + \frac{z_{\text{in}}}{\gamma} + \frac{w_{\text{shaft net in}}}{g} - h_L \]
Example 5.26 Energy – Effect of Nonuniform Velocity Profile

The small fan shown in Figure E5.26 moves air at a mass flowrate of 0.1 kh/min. Upstream of the fan, the pipe diameter is 60 mm, the flow is laminar, the velocity distribution is parabolic, and the kinetic energy coefficient, $\alpha_1$, is equal to 2.0. Downstream of the fan, the pipe diameter is 30 mm, the flow is turbulent, the velocity profile is quite uniform, and the kinetic energy coefficient, $\alpha_2$ , is equal to 1.08. If the rise in static pressure across the fan is 0.1 kPa and the fan motor draws 0.14 W, compare the value of loss calculated: (a) assuming uniform velocity distributions, (2) considering actual velocity distribution.
Example 5.26 Energy – Effect of Nonuniform Velocity Profile

\[ D_2 = 30 \text{ mm} \]

Turbulent flow

Section (2)
\[ \alpha_2 = 1.08 \]

Control volume

\[ D_1 = 60 \text{ mm} \]

Section (1)
\[ \alpha_1 = 2.0 \]

Laminar flow
\[ \dot{m} = 0.1 \text{ kg/min} \]
The energy equation for non-uniform velocity profile…….. 

\[
\frac{p_2}{\rho} + \frac{\alpha_2 V_2^2}{2} + g z_2 = \frac{p_1}{\rho} + \frac{\alpha_1 V_1^2}{2} + g z_1 + w_{\text{shaft net/in}} - \text{loss}
\]

\[
\text{loss} = w_{\text{shaft net/in}} - \left( \frac{p_2 - p_1}{\rho} \right) + \frac{\alpha_1 V_1^2}{2} - \frac{\alpha_2 V_2^2}{2}
\]

\[
w_{\text{shaft net/in}} = \frac{\text{power to fan motor}}{\dot{m}} = \frac{(0.14 W)[(1N \cdot m/s) / W]}{0.1 \text{kg/min}} (60 \text{s/min}) = 84.0 N \cdot m / \text{kg}
\]

\[
\bar{V}_1 = \frac{\dot{m}}{\rho A_1} = ... = 0.479 \text{m/s} \quad \bar{V}_2 = \frac{\dot{m}}{\rho A_2} = ... = 1.92 \text{m/s}
\]
Example 5.26  Solution\(^{1/2}\)

\[
\text{loss} = w_{\text{shaft net/in}} - \left( \frac{p_2 - p_1}{\rho} \right) + \frac{\alpha_1 \overline{V}_1^2}{2} - \frac{\alpha_2 \overline{V}_2^2}{2}
\]

\[
= 0.975 \text{N} \cdot \text{m/kg} (\alpha_1 = \alpha_2 = 1)
\]

\[
\text{loss} = w_{\text{shaft net/in}} - \left( \frac{p_2 - p_1}{\rho} \right) + \frac{\alpha_1 \overline{V}_1^2}{2} - \frac{\alpha_2 \overline{V}_2^2}{2}
\]

\[
= 0.940 \text{N} \cdot \text{m/kg} (\alpha_1 = 2, \alpha_2 = 1.08)
\]
Example 5.28 Energy – Fan Performance

For the fan of Example 5.19, show that only some of the shaft power into the air is converted into a useful effect. Develop a meaningful efficiency equation and a practical means for estimating lost shaft energy.
Example 5.28 Solution $1/2$

\[
\frac{p_2}{\rho} + \frac{V_2^2}{2} + g z_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2} + g z_1 + w_{\text{shaft net in}} - \text{loss} \quad (1)
\]

Useful effect = \( w_{\text{shaft net/in}} - \text{loss} \)

\[
= \left( \frac{p_2}{\rho} + \frac{V_2^2}{2} + g z_2 \right) - \left( \frac{p_1}{\rho} + \frac{V_1^2}{2} + g z_1 \right) \quad (2)
\]

Efficiency \( \eta = \frac{w_{\text{shaft net in}} - \text{loss}}{w_{\text{shaft net in}}} \quad (3) \)

\[w_{\text{shaft net in}} = +U_2 V_{\theta 2} \quad (4)\]
Example 5.28 Solution\(^{2/2}\)

(2)+(3)+(4)

\[
\eta = \{(p_2 / \rho) + (V_2^2 / 2) + gz_2 \} - \{(p_1 / \rho) + (V_1^2 / 2) + gz_1 \} / U_2 V_{\theta_2}
\]

(2)+(4)

\[
\eta = U_2 V_{\theta_2} - [\{(p_2 / \rho + V_2^2 / 2 + gz_2 \} - (p_1 / \rho + V_1^2 / 2 + gz_1)]
\]